Mixing Effects in continuously-modulated forward roll coating

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Two-dimensional steady laminar fluid flows in bounded domains often feature regions of recirculation in which fluid is trapped and separated from the rest of the flow. Under steady conditions these recirculations persist since fluid may not cross their bounding streamlines; 'tracer' particles placed within a recirculation would execute non-chaotic closed orbits (Aref 1984). However, there are several ways in which such steady flows can be modulated in time to produce chaotic fluid motion and an interaction between fluid within a recirculation and that outside it. Two of the most popular geometries studied are the journal bearing (e.g. Kaper & Wiggins 1993) and the rectangular double-lid-driven cavity (e.g. Chien *et al.* 1986). In both of these systems the flow is driven by the tangential motion of two boundaries, i.e. the rotation of the inner and outer cylinders or the parallel translational motion of the two lids, and chaotic flow can be achieved by varying the speeds of these boundaries periodically in so-called modulation 'protocols'. Protocols can be discontinuous, involving the alternating steady motion of one boundary then the other, or continuous, for example the sinusoidal modulations of Kaper & Wiggins (1993). In both cases the geometry of the domain remains the same.

An alternative means of generating chaos is to modulate the shape of the domain through *non*-tangential motion of one or more boundaries. This was first done in the 'baffled cavity flow' by Jana *et al.* (1994). The flow consisted of a rectangular driven cavity into which a number of baffles were periodically inserted and withdrawn. More recently, Fin & Cox (2001) studied the stirring effect of a circular cylindrical rod moving on a cycloidal path through a static cylindrical container of fluid. A further example is provided by Horner *et al.* (2002), who studied the flow in a three-sided rectangular cavity whose fourth side was open to an annular channel; the perturbation to the steady flow was caused by the rotating inner cylinder, whose surface was ribbed.

The aim of this work was to apply the ideas developed in the cited literature to an inlet-flooded forward roll coating system. The flow features aspects of both the fixed-domain modulated-speed flows and the geometrically-modulated flows, since it involves a flow between two solid rotating cylinders and includes a free surface whose position and shape over time in response to the speeds of the cylinders. A sketch of the geometry is given in figure 1, while figure 2 shows the steady-state flow structures seen in the recirculation zone.

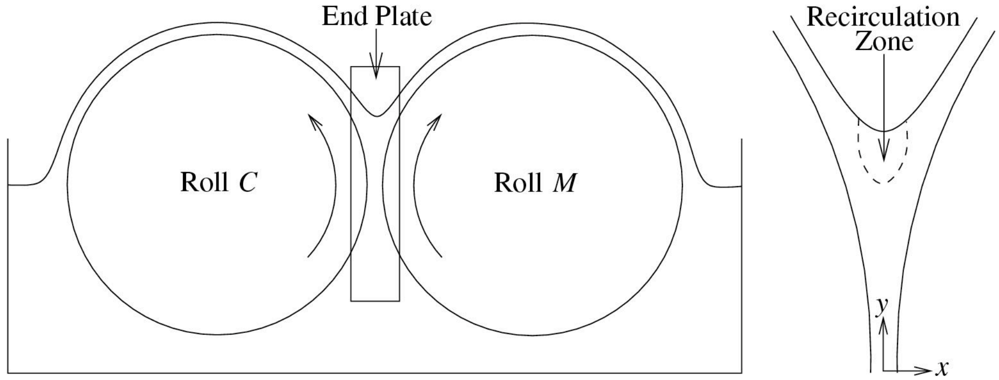

Figure 1: Schematic showing a cross-section of the rig, and the computational domain. Roll C has a constant speed, while the speed of roll M is modulated periodically.

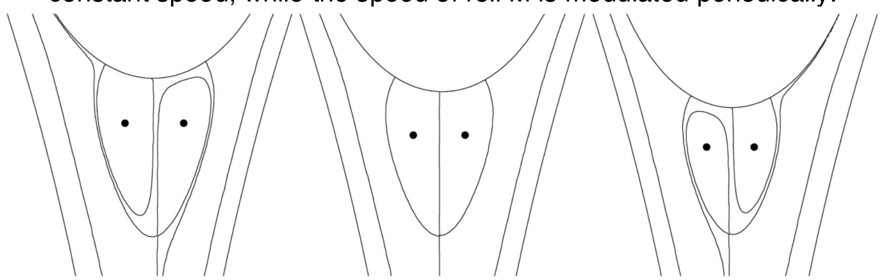

Figure 2: Steady-state streamlines. The black circles show the positions of elliptic stagnation points. In the left-hand picture, the left-hand roll is moving faster; in the right-hand picture the right-hand roll is the faster one. In the middle picture the roll speeds are equal.

A simple and efficient means of modulating the speed of roll M was provided by driving the roll through a universal (or 'Hooke's') joint, as shown in figure 3. This joint has a well-characterised behaviour described by the expression

$$S(t) = \frac{U_M}{U_C} = \frac{\cos\theta}{1 - \sin^2\theta\cos^2\Omega t}$$

where Ω is the angular velocity of the drive shaft, and t is time, and S is the speed ratio of the modulated to unmodulated roll. This expression enables simulations using the same modulation to be performed.

The numerical simulations were undertaken by solving the time-dependent Navier-Stokes equations using a Galerkin finite element formulation with the following boundary conditions:

- Roll surfaces: no slip condition using instantaneous value of roll speed.
- Free surface: zero shear stress and surface-tension-balanced normal stress, plus a time-dependent kinematic condition matching the normal components of the free surface and fluid velocities.

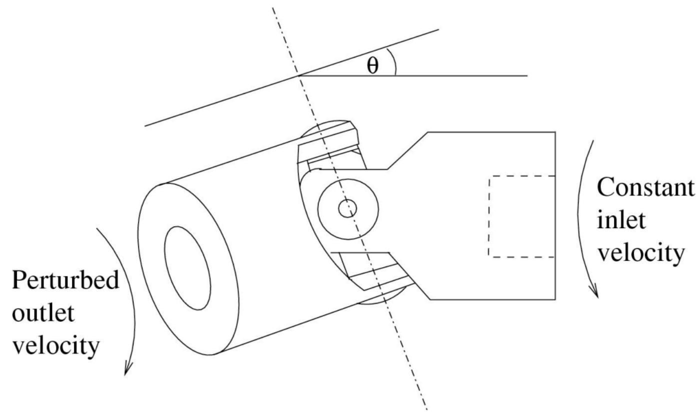

Figure 3: Hooke's Joint linkage used to modulate roll speed.

- Outflow: the 'free boundary' condition of Papanastasiou *et al.* (1992)
- Inflow: matched to lubrication theory

The paths of passive 'tracer' particles were calculated from the resulting velocity fields using a 4th order Runge-Kutta scheme.

An example of a comparison between a simulation and an experiment involving dye injection is given in figure 4, where time increases by 1 second per frame in the vertical sense. The figure shows very good agreement, and demonstrates that the simulation captures the behaviour of the dye blob under the Hooke's joint modulation. Figure 5 offers a closer look at the mechanism of fluid exchange between the recirculation region and the surrounding fluid. The figure shows one period of the modulation, starting from a steady state with the roll speeds equal. In this initial state, the recirculation region is uniformly 'populated' by a large number of tracer particles. The subsequent frames show (with time increasing in the horizontal sense) how the flow develops. As the free surface moves downwards, tracers are expelled from the recirculation region along the right-hand film surface. A much smaller number leave the eddy to the left. After the free surface has reached its lowest point and it moves back upwards, ambient fluid is drawn into the eddy region – see the white 'finger' of surrounding fluid entering and progressing down the centre of the eddy zone. This behaviour is very similar to the transport mechanism seen in a rigid geometry by Horner *et al.* (2002) and should therefore be amenable to the same analysis in terms of 'turnstile lobes'. At the end of the period, the white patch indicates that a proportion of the fluid originally in the eddy has been ejected and replaced with 'fresh' fluid.

Figure 6 attempts to give a more quantitative picture of the fluid exchange process by plotting the percentage of particles initially in the recirculation zone which remain in the domain as time progresses. The different curves correspond to different Hooke's joint angles, and therefore different modulation amplitudes (amplitude increases with θ).

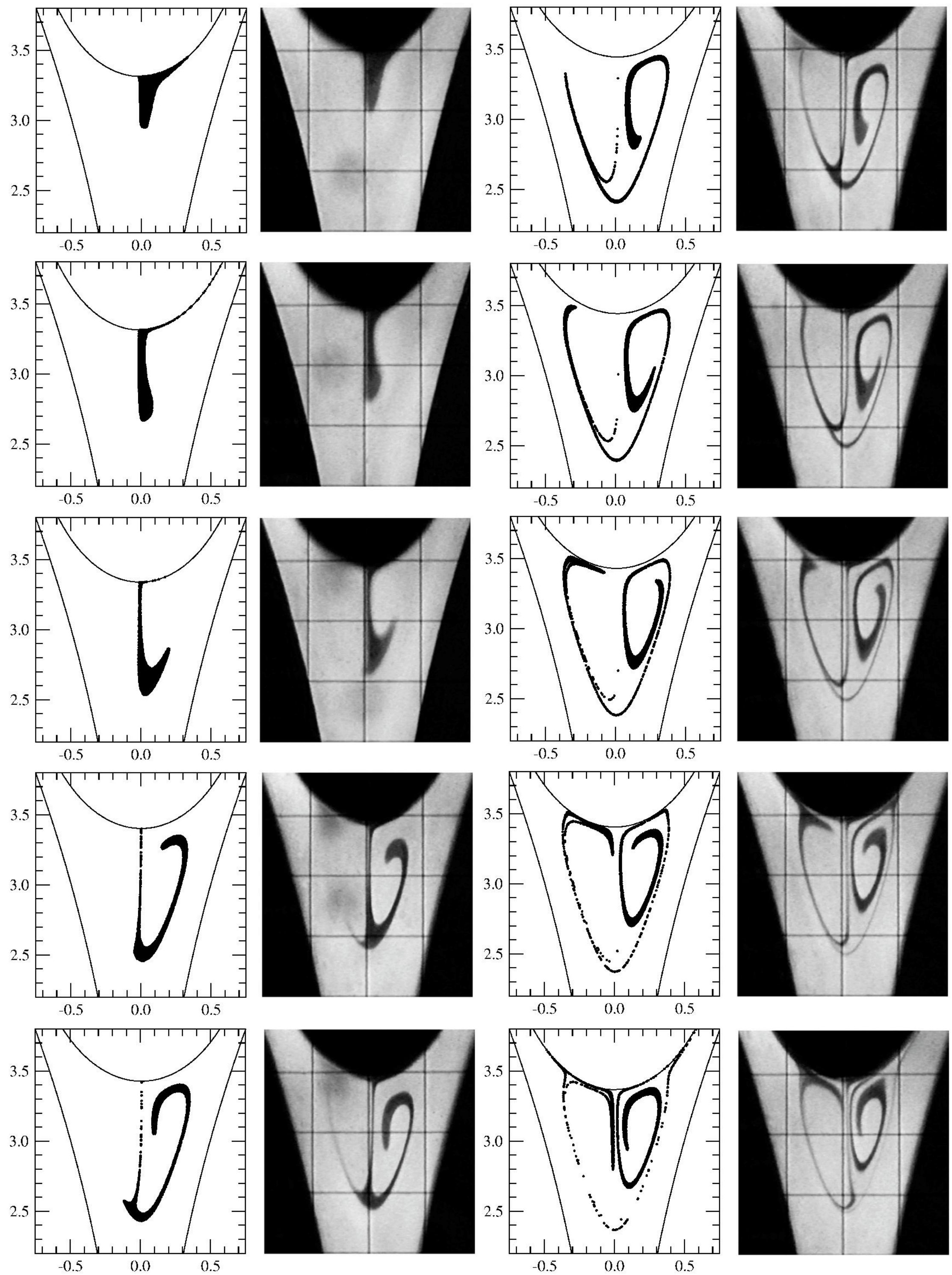

Figure 4: Comparison of simulations and experiments involving an injected dye blob. Time increases by 1 second per frame in the vertical sense. The angle of the Hooke's joint was 30°.

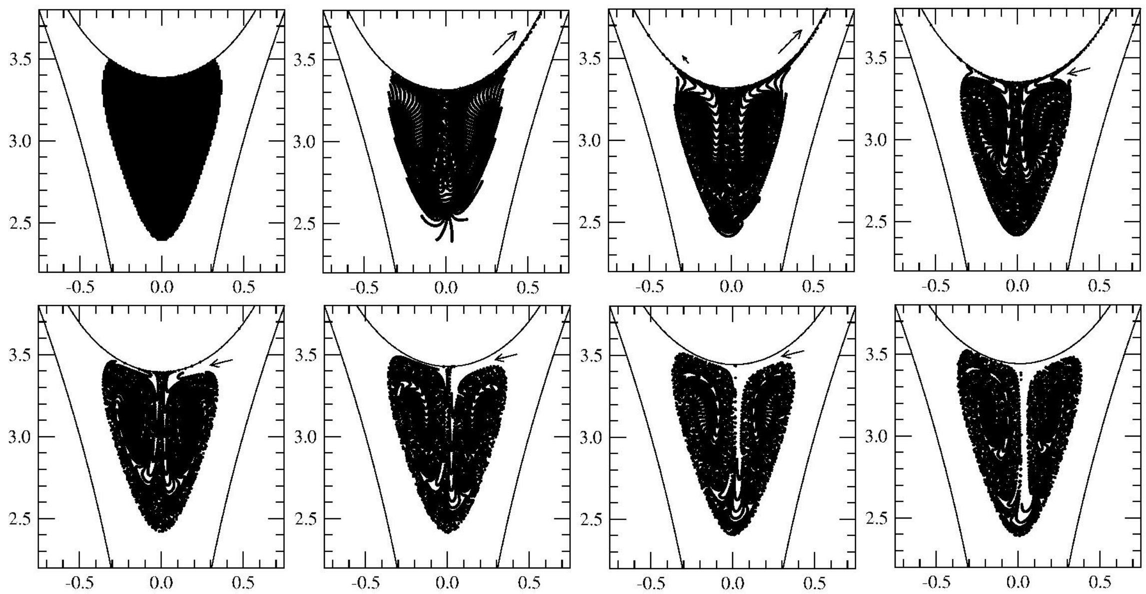

Figure 5: Numerical simulation of approximately 6000 points initially uniformly distributed throughout the recirculation zone. The Hooke's joint angle was 30°, giving a modulation amplitude of 15%.

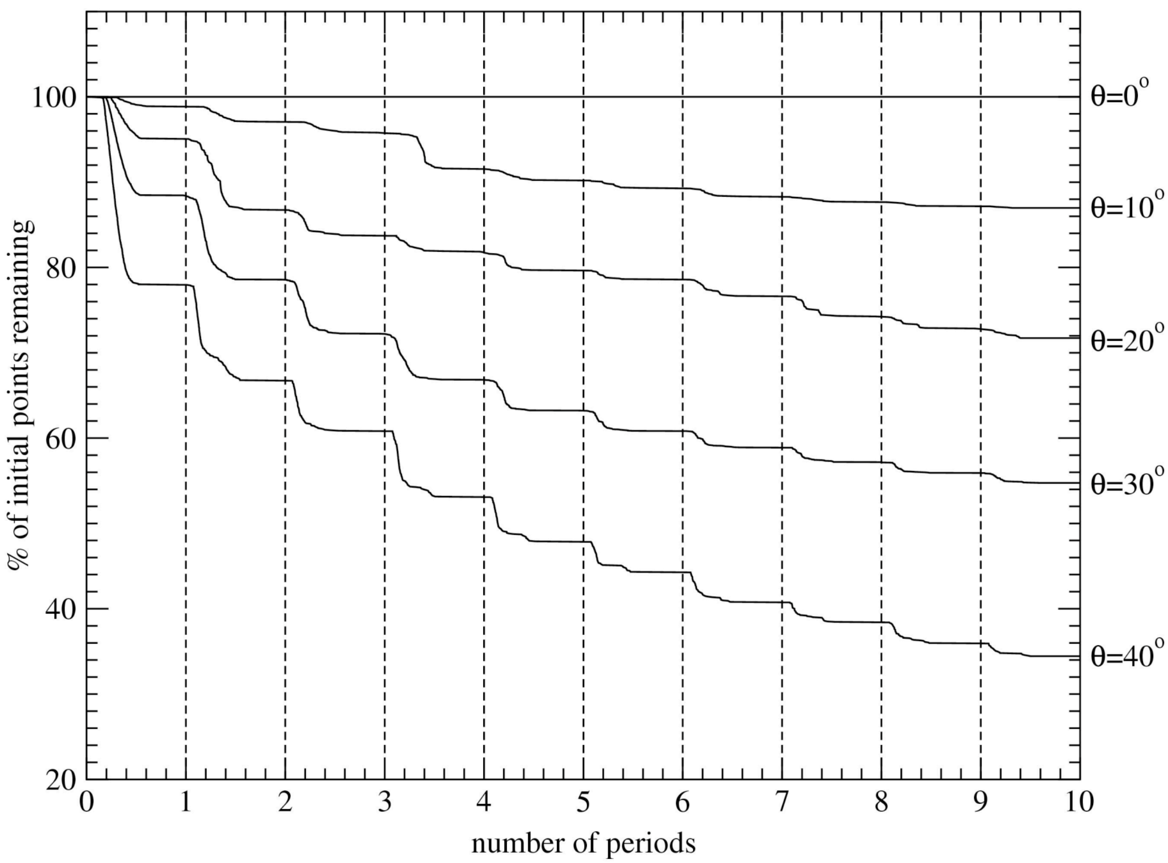

Figure 6: Percentage of initial points remaining inside the domain as a function of time for different Hooke's joint angles.

The results show the ejection of tracers in the first quarter of each period, during which the free surface moves inwards. The curves also indicate that – as one might expect – the rate of fluid exchange increases with increasing modulation amplitude. Each curve levels off, and for this speed ratio (S=1) there is always a greater or lesser residual core of unmixed fluid which remains in the domain. Higher speed ratios seem to respond more strongly to modulations, however. For S=1.5, almost all fluid is exchanged when the flow is subjected to the 15% amplitude modulation provided by the Hooke's joint inclined at 30°, while for S=1 more than half remains umixed. Another important factor controlling the mixing efficiency is the modulation frequency, and work is currently underway to explore this effect in the above free surface flow.

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