Mathematical modelling of curtain coating

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Abstract

We present a simple mathematical model for the fluid flow in the curtain coating process, exploiting the small aspect ratio, and examine the model in the large-Reynolds-number limit of industrial interest. We show that the fluid is in free fall except for a region close to the substrate, but find that the model can not describe the turning of the curtain onto the substrate. We find that the inclusion of a viscous bending moment close to the substrate allows the curtain to "turn the corner".

1 The industrial method of curtain coating

Curtain coating is an industrial process, traditionally used to coat photographic film, which is now beginning to be used by the paper industry. A reservoir of coating mix, typically an aqueous solution containing 20-50% solids and surfactants, is formed into a curtain of fluid using either a slot or a slide as shown in Figure 1. This curtain falls under gravity until it hits the substrate or "web" to be coated, which is conveyed quickly underneath. The coated substrate is passed through driers to evaporate off the water and thus the finished paper is left with a, hopefully uniform, solid coating. The coater machines have edge guides with water running in them to fix the width of the curtain as, without them, the curtain is observed to "neck in" laterally. In the paper industry this technology is run at much higher web speeds, and with fluids of very different rheologies, than in the photographic industry.

1.1 Advantages and disadvantages of curtain coating

Methods currently used to coat paper include blade coating and air-knife coating, where a thick layer of fluid is applied and scraped off to leave the desired thickness. This, however, places a constraint on the speed of coating since, when operated at high speeds,

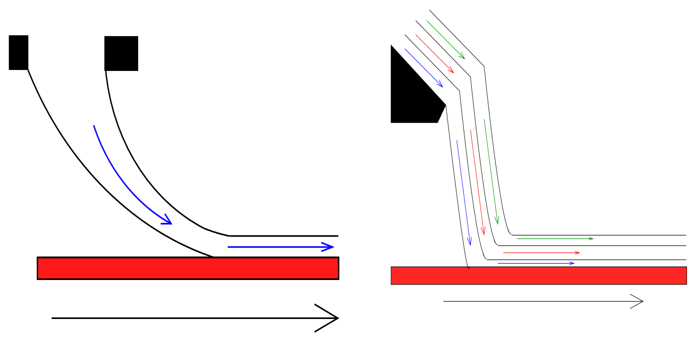

Figure 1: The slot and slide coaters.

it subjects the substrate to a high shear, which can cause breakages. Curtain coating does not produce this high shear, and so the substrate can be moved at higher speeds.

Other advantages of curtain coating over blade or air-knife coating include that a low coat weight (i.e. a small quantity of solids) may be applied, which reduces costs, and that contoured paper may be coated successfully, since the method lays down a uniform solid coating.

However, curtain coating also has disadvantages when compared with blade or air-knife coating. For example, it can be hard to form a stable curtain if, for instance, the flow rate is too low; in this case the curtain can detach from the edge guides or break into individual streams.

There are many problems inherent to the zone where the curtain meets the substrate (the impingement zone). They include air entrainment, skip coating and the formation of a heel. Air entrainment occurs when the substrate is moving too fast, whereupon the fluid in the curtain does not wet the substrate properly and defects such as bubbles, pin holes and streaks appear. If the substrate is moving much too fast then skip coating can occur, where the mix "bounces" off the web. If the substrate is moving too slowly the curtain can "bulge" out away from the direction of motion where it meets the substrate, forming a heel. This also causes coating defects similar to air entrainment, and can cause recirculation, and so degradation, of the mix. Details of these may be found in [1].

1.2 Previous mathematical models

One of the first papers written on the subject of curtain coating is by Brown [2]. This is a mainly experimental introduction to the basic dynamics of a falling sheet, but includes a mathematical appendix by GI Taylor deriving the equation of motion. Brown finds empirically that the fluid is in ballistic free fall for the majority of the curtain.

A general literature survey on coating methods is given in [3]. This includes work on more general aspects of coating, such as the flow of a thin film, the boundary layer along a moving wall, as in the Sakiadis model [4], and the dynamic wetting line as well as areas more closely related to curtain coating such as the stability of the curtain, the propagation of waves in the fluid on the slide and the flow in the curtain.

Theoretical and experimental stability analyses have been performed, as in [5] and [2], which find the curtain to be stable if any disturbances in the curtain are "swept away" downstream before they have a chance to grow. This equates mathematically to the condition that the Weber number (which represents a balance between inertia and surface tension forces)

$$\mathrm{We} = \frac{\rho q V}{2\gamma} \tag{1}$$

is greater than unity, where ρ is the density, q is the volumetric liquid flux, V is the local fluid speed and γ is the surface tension.

2 A steady-state model of the curtain

We derive steady-state equations for the curtain using an arc-length-based coordinate system. Since in all operating regimes the length of the curtain is much greater than the thickness, we have a small aspect ratio $\epsilon = h_0/l$, where h_0 is the slot width and l is the (as yet unknown) length of the curtain. Hence, by analogy with the Trouton model of extensional viscous flow [6], we may assume uniform flow across the thickness of the curtain.

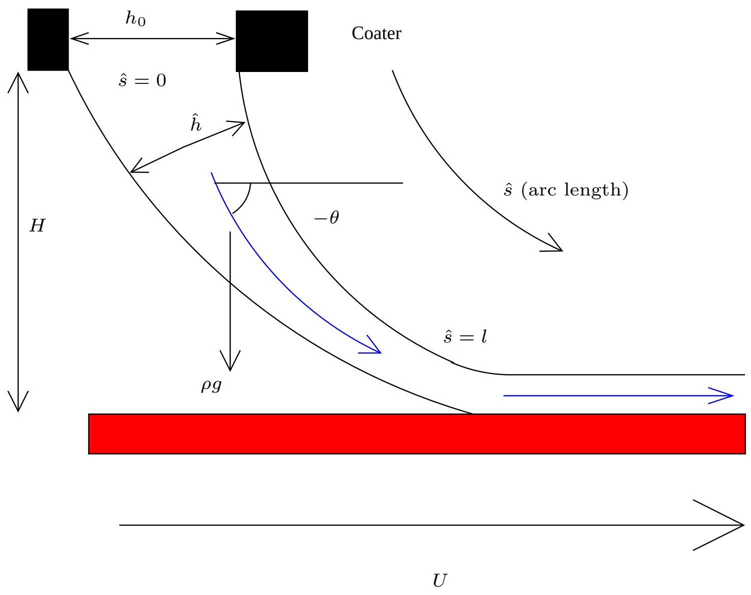

Figure 2: The coordinates in the curtain.

2.1 Derivation of equations

We model the curtain in the simplest possible way, taking a cross section through the plane normal to the substrate, far away from any edge effects and also assume the process to be in a steady-state. Our coordinates are shown in Figure 2. We parametrise with respect to arc-length $\hat{s}$ along the centre-line of the curtain and take θ to be the angle, measured upwards, that this line makes with the horizontal. We denote the thickness of the curtain by $\hat{h}$, the velocity of the fluid along the centre-line by $\hat{u}$, the tension due to viscosity by $\hat{T}$ and the *a priori* unknown length of the curtain by l. This gives us a free boundary problem, similar to a contact problem in elasticity.

We build our model using conservation of mass and momentum. Assuming the fluid in the curtain to be incompressible, conservation of mass immediately gives us

$$\hat{h}\hat{u} = q, \tag{2}$$

where q is the constant volumetric flux supplied through the nozzle of the coater. With ρ being the (assumed constant) density of the fluid and g being gravity, conservation of momentum yields

$$\frac{d}{d\hat{s}}\left(\hat{T}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}\right) + \rho g\hat{h}\begin{pmatrix}0 \\ -1\end{pmatrix} = \frac{d}{d\hat{s}}\left(\rho\hat{h}\hat{u}^2\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}\right). \tag{3}$$

Here the first term on the left hand side is the tension force, the second term is the gravity force and the right hand side is the momentum flux. If we now dot with the tangent vector $(\cos\theta, \sin\theta)^T$, we have

$$\hat{T}' - \rho q\hat{u}' = \rho g\hat{h}\sin\theta, \tag{4}$$

representing conservation of tangential momentum, and dotting with the normal vector $(\sin\theta, -\cos\theta)^T$ similarly gives us

$$\left(\hat{T} - \rho q\hat{u}\right)\theta' = \rho g\hat{h}\cos\theta, \tag{5}$$

representing conservation of normal momentum, where $' \equiv d/d\hat{s}$.

This gives us three equations (2), (4), (5) for the four unknowns θ, $\hat{T}$, $\hat{u}$ and $\hat{h}$, so we need a further constitutive relationship to tell us about the properties of the fluid. In this simplistic model, we assume that the fluid is Newtonian with constant viscosity μ, i.e.

$$\hat{T} = 4\mu\hat{h}\hat{u}', \tag{6}$$

as in the Trouton model [6]. We note that other properties of the fluid such as surface tension or viscoelasticity could be incorporated by modifying this constitutive relationship.

We now need to apply boundary conditions. Having prescribed the flux through the coater nozzle our other boundary conditions are that at the contact point the fluid is moving at the substrate speed, and that at the coater head we prescribe the velocity of the fluid. The new unknown l in the problem is determined by the condition that the height H of the coater above the substrate is known. These conditions equate mathematically to

$$\begin{aligned} \theta = \theta_0 \quad &\text{at} \quad \hat{s} = 0, && (7)\\ \hat{u} = U \quad &\text{at} \quad \hat{s} = l, && (8)\\ \hat{u} = q/h_0 \quad &\text{at} \quad \hat{s} = 0, && (9)\\ \int_0^l \sin\theta \, d\hat{s} \quad &= \quad -H. && (10) \end{aligned}$$

Notice that we specify the angle θ at the top of the curtain but not at the substrate. We will show below in Section 3 that a further term must be incorporated into (3) if we wish to impose tangential contact between the curtain and the substrate. Note also that the thickness of the coating on the substrate is given by

$$h_1 = \hat{h}(l) = \frac{q}{U}. \tag{11}$$

2.2 Nondimensionalisation

We now rearrange our equations and nondimensionalise using

$$\hat{h} = h_0 h, \quad \hat{u} = Uu, \quad \hat{s} = ls, \quad \hat{T} = \frac{4\mu q}{l} T$$

to give

$$\begin{aligned} uh &= v, && (12)\\ u' &= Tu, && (13)\\ T' &= \mathrm{Re}\, Tu + \frac{\mathrm{Re}}{\mathrm{Fr}^2}\frac{\sin\theta}{u}, && (14)\\ \theta' &= \frac{1}{\mathrm{Fr}^2}\frac{\cos\theta}{u\left(\frac{T}{\mathrm{Re}} - u\right)}, && (15) \end{aligned}$$

where

$$\mathrm{Re} = \frac{\rho U l}{4\mu} \tag{16}$$

is the Reynolds number, which represents a balance between inertial and viscous effects,

$$\mathrm{Fr} = \frac{U}{\sqrt{gl}} \tag{17}$$

is the Froude number, which represents a balance between inertial and gravity effects, and

$$v = \frac{q}{Uh_0} = \frac{h_1}{h_0} \tag{18}$$

is the draw ratio. Note here that we are nondimensionalising with the unknown value of l and so finding Re and Fr is part of the problem: we will have to "back them out". The boundary conditions become

$$\theta = \theta_0,\ u = v \quad \text{at} \quad s = 0, \tag{19}$$

$$u = 1 \quad \text{at} \quad s = 1, \tag{20}$$

$$\int_0^1 \sin\theta \, ds = -\frac{H}{l}. \tag{21}$$

2.3 Immediate observations

We first note that once we have solved these equations to find $\theta(s)$ we can then solve $dx/ds = \cos\theta$, $dy/ds = \sin\theta$ to give the centre-line of the curtain in xy coordinates. Here we have nondimensionalised x and y with l as we did with s. We are then easily able to draw the shape of the curtain since the two free surfaces of the curtain are given by $(x \mp \epsilon h \sin\theta/2, y \pm \epsilon h \cos\theta/2)$.

By considering (13) and (14) we obtain the following constant of the motion

$$(T/\text{Re} - u)\cos\theta = \text{constant} = C, \tag{22}$$

which is equivalent to a horizontal force balance. In particular, if $\theta = \pm\pi/2$ at any point (e.g. in a slot coater) then $\theta \equiv \pm\pi/2$. Moreover if $\theta \neq \pm\pi/2$ initially, then it is impossible for θ to cross $\pm\pi/2$ at any point.

2.4 Solution in a typical operating regime

We have three dimensional parameters Re, Fr and v, and so we look for an asymptotic limit. Typical operating parameters could be Re $\approx$ 1000–10000 and Fr ≈ 10, so we take the asymptotic limit Re $\gg 1$ with Fr $= O(1)$.

We take the simplest case $\theta_0 = -\pi/2$ so that the curtain exits the slot vertically, and solve (13)-(15) in the limit Re $\to \infty$ with condition (19) to find

$$T = \frac{1}{\text{Fr}^2 u}, \quad u = \sqrt{2s/\text{Fr}^2 + v}, \quad \theta = -\frac{\pi}{2}. \tag{23}$$

This is precisely a ballistic free fall velocity and we notice that, in this limit, the substrate has no effect on the flow in the curtain. However, the solution (23) does not satisfy the boundary condition $u(1) = 1$, so we look for a boundary layer near $s = 1$ across which the velocity adjusts. We find we must rescale T and s such that $T = \text{Re}\,\bar{T}$, and $s = 1 + \bar{s}/\text{Re}$ to give

$$\text{Re}\, u' = \text{Re}\, \bar{T} u, \tag{24}$$

$$\text{Re}^2\, \bar{T}' = \text{Re}^2\, \bar{T} u + \frac{\text{Re}}{\text{Fr}^2}\frac{\sin\theta}{u}, \tag{25}$$

$$\text{Re}\, \theta' = \frac{1}{\text{Fr}^2}\frac{\cos\theta}{u\left(\bar{T} - u\right)}. \tag{26}$$

Taking the limit Re $\to \infty$ we see that $\theta' = 0$ and, since we require $\theta \to -\pi/2$ as $\bar{s} \to -\infty$ to match with the flow outside the boundary layer, we must have

$$\theta \equiv -\frac{\pi}{2}, \tag{27}$$

that is the curtain can never turn around onto the substrate in this one-dimensional model. The remaining equations in this limit are

$$\bar{T}' = \bar{T} u, \tag{28}$$

$$u' = \bar{T} u, \tag{29}$$

with boundary and matching conditions

$$u = 1 \quad \text{on} \quad \bar{s} = 0, \tag{30}$$

$$u \to u_*, \ \bar{T} \to 0 \quad \text{as} \quad \bar{s} \to -\infty, \tag{31}$$

where

$$u_* \;=\; \sqrt{2/\mathrm{Fr} + v^2}, \tag{32}$$

is the impingement speed into the boundary layer. Now, we notice from equations (28) and (29) that $\bar{T} - u = \text{const} = -u_*$, using (31). Thus we can reduce (28) and (29) to

$$u' = (u - u_*)u, \tag{33}$$

and integrate to give the solution

$$u \;=\; \frac{u_*}{1 + (u_* - 1)\, e^{u_* \bar{s}}}, \tag{34}$$

$$\bar{T} \;=\; \frac{u_*\,(1 - u_*)\, e^{u_* \bar{s}}}{1 + (u_* - 1)\, e^{u_* \bar{s}}}. \tag{35}$$

If we now find a composite expansion of the outer and inner solutions, so that we represent the solutions in both regions in one expression, this gives us

$$u \;=\; \sqrt{2s/\mathrm{Fr}^2 + v^2} + \frac{u_*}{1 + e^{u_* \mathrm{Re}(s-1)}\,(u_* - 1)} - u_*, \tag{36}$$

$$T \;=\; \frac{1}{\mathrm{Fr}^2 \sqrt{2s/\mathrm{Fr}^2 + v^2}} + \mathrm{Re}\, \frac{u_*\,(1 - u_*)\, e^{u_* \mathrm{Re}(s-1)}}{1 + e^{u_* \mathrm{Re}(s-1)}\,(u_* - 1)}. \tag{37}$$

We plot these solutions using $\mathrm{Re} = 1000$, $\mathrm{Fr} = 10$ and $v = 0.6$ in Figure 3. We see that the velocity increases slowly until it reaches the boundary near $s = 1$ where it increases sharply to the substrate speed. We also see that the tension is virtually zero and decreasing until the boundary layer is reached where it too increases sharply as the substrate exerts influence.

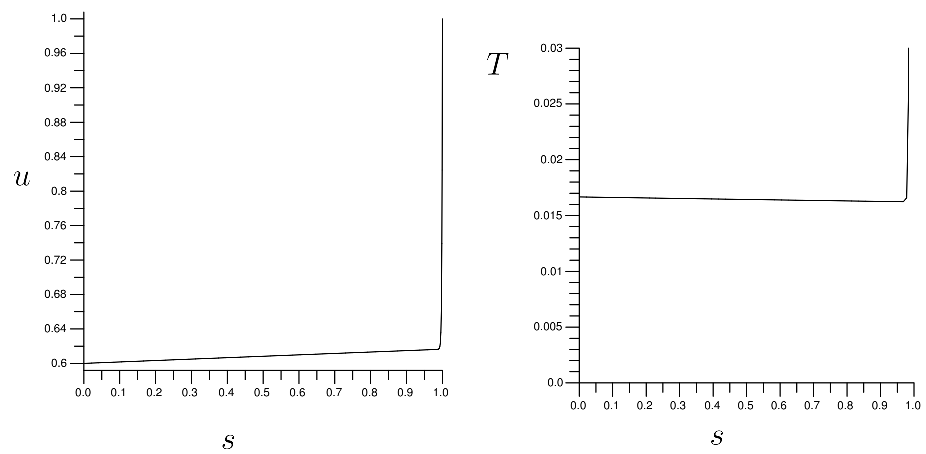

Figure 3: The composite expansions (36) and (37) for u and T, taking $\mathrm{Re} = 1000$, $\mathrm{Fr} = 10$ and $v = 0.6$ in both.

3 Including a bending moment

In Section 2 we showed that our model does not allow the curtain to “turn the corner” onto the substrate in the inertia-dominated case. Physically, however, it is clear that the curtain does turn onto the substrate, and thus our model must be over-simplified. We now include a bending moment term which, we will see, allows the corner to be turned.

Like an elastic solid, a bending moment exists in a fluid due to the transverse and longitudinal forces acting upon it. This bending moment is always present, but is often so small as to be negligible. We will investigate whether this bending moment is important in the process of curtain coating.

3.1 Derivation of equations

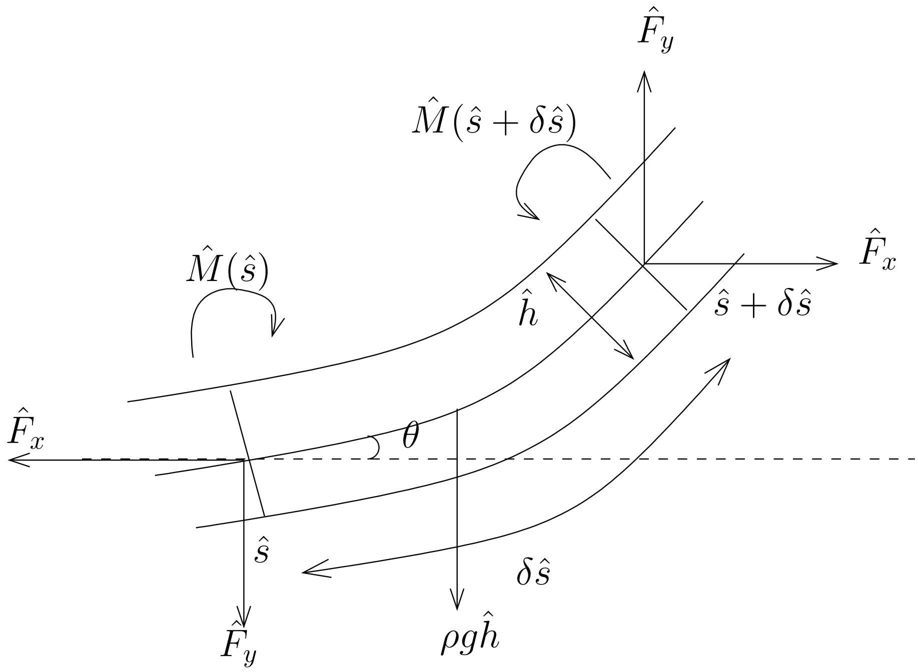

Figure 4: The forces on a small segment of the curtain.

We consider the situation shown in Figure 4, where we take $\hat{F}_x$ and $\hat{F}_y$ to be the horizontal and vertical forces acting on the curtain (so we have $\hat{T} = \hat{F}_x \cos\theta + \hat{F}_y \sin\theta$) and $\hat{M}$ to be the bending moment. Conservation of mass and momentum then give

$$\hat{u}\hat{h} = q, \tag{38}$$

$$\hat{F}_x' = (\rho q \hat{u} \cos\theta)', \tag{39}$$

$$\hat{F}_y' = \frac{\rho g q}{\hat{u}} + (\rho q \hat{u} \sin\theta)', \tag{40}$$

and a balance of moments gives

$$\hat{M}' = \hat{F}_x \sin\theta - \hat{F}_y \cos\theta, \tag{41}$$

where $' \equiv d/d\hat{s}$. We also have

$$\hat{F}_x \cos\theta + \hat{F}_y \sin\theta = 4\mu\hat{h}\hat{u}', \tag{42}$$

from the constitutive relationship, and, for a Newtonian viscous fluid,

$$\hat{M} = -\frac{\hat{h}^3}{3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\mu\theta'\right) \tag{43}$$

$$= -\frac{\mu\hat{u}\hat{h}^3}{3}\theta'', \tag{44}$$

assuming steady-state. Details on the derivation of (43) can be found in [7].

These equations add an extra derivative in θ which means we may now prescribe θ at both the coater nozzle and the contact point on the substrate.

3.2 Nondimensionalisation

We replace $\hat{h}$ with $q/\hat{u}$ and nondimensionalise using

$$\hat{u} = Uu, \quad \hat{s} = ls, \quad \hat{F}_{x,y} = \frac{4\mu q}{l}F_{x,y}, \quad \hat{M} = \left(\rho^2\mu q^5\right)^{1/3} M.$$

This gives us,

$$(F_x - \mathrm{Re}\, u\cos\theta)' = 0, \tag{45}$$

$$(F_y - \mathrm{Re}\, u\sin\theta)' = \frac{\mathrm{Re}}{\mathrm{Fr}^2 u}, \tag{46}$$

$$M = -\frac{\delta^2}{3u^2}\theta'', \tag{47}$$

$$F_x\sin\theta - F_y\cos\theta = \delta\mathrm{Re}\, M', \tag{48}$$

$$F_x\cos\theta + F_y\sin\theta = \frac{u'}{u}. \tag{49}$$

where Re and Fr are given by (16) and (17) and

$$\delta = \frac{\left(\mu q^2/\rho\right)^{1/3}}{lU}. \tag{50}$$

measures the importance of the bending moment. Note that, by letting $\delta \to 0$, after some manipulation we obtain our original equations neglecting the bending moment, as found in section 2. It is also helpful to define

$$\alpha = \delta\,\mathrm{Re} = \frac{1}{4}\left(\frac{\rho q}{\mu}\right)^{2/3}. \tag{51}$$

3.3 Solution in a typical operating regime

We now have four parameters Re, Fr, δ and v, and so we again look for an asymptotic limit. Typical operating conditions could be such that Re $\approx$ 1000–10000, Fr ≈ 10 and α is an order one constant, and so we solve this system in the limit $\delta \ll 1$, so that $\mathrm{Re}/\alpha \gg 1$, and $\mathrm{Fr} = O(1)$. Note here that α being order one means that the boundary layers in u and θ are of the same order. We again take $\theta_0 = -\pi/2$ for simplicity and solve the equations to find (as $\mathrm{Re} \to \infty$)

$$F_x = M = 0, \tag{52}$$

$$\theta = -\frac{\pi}{2}, \tag{53}$$

$$u = \sqrt{2s/\mathrm{Fr}^2 + v^2}, \tag{54}$$

$$F_y = -\frac{1}{\mathrm{Fr}^2\left(2s/\mathrm{Fr}^2 + v^2\right)}, \tag{55}$$

as before. However, this again does not satisfy the boundary conditions at the contact point on the substrate. Therefore we look for a boundary layer near here by rescaling $s = 1 + \delta\bar{s}$, $F_{x,y} = \mathrm{Re}\,\bar{F}_{x,y}$ to give

$$\begin{aligned}
(\bar{F}_x - u\cos\theta)' &= 0, && (56)\\
\frac{(\bar{F}_y - u\sin\theta)'}{\delta} &= \frac{1}{\mathrm{Fr}^2 u}, && (57)\\
M &= -\frac{\theta''}{3u^2}, && (58)\\
\bar{F}_x\sin\theta - \bar{F}_y\cos\theta &= M', && (59)\\
\mathrm{Re}\,\left(\bar{F}_x\cos\theta + \bar{F}_y\sin\theta\right) &= \frac{1}{\delta}\frac{u'}{u}. && (60)
\end{aligned}$$

Letting $\mathrm{Re}\,\delta = \alpha = O(1)$ and $\delta \to 0$ we find

$$\begin{aligned}
\bar{F}_x &= u\cos\theta, && (61)\\
\bar{F}_y &= u\sin\theta + u_*, && (62)\\
M' &= -u_*\cos\theta, && (63)\\
\theta'' &= -3u^2 M, && (64)\\
\frac{1}{\alpha}\frac{u'}{u} &= u + u_*\sin\theta. && (65)
\end{aligned}$$

where $u_* = u(1)$ is again the impingement speed into the boundary layer. We let $N = \theta'$ so that

$$N' = -3Mu^2, \qquad (66)$$

which reduces the problem to a system of first order ordinary differential equations, and the equations for $\bar{F}_x$ and $\bar{F}_y$ decouple. We rescale $u = u_*\tilde{u}$, $N = u_*\tilde{N}$, $\bar{s} = \tilde{s}/u_*$ to remove u_* from the problem, to give

$$\begin{aligned}
M' &= -\cos\theta, && (67)\\
\frac{\tilde{u}'}{\alpha\tilde{u}} &= \tilde{u} + \sin\theta, && (68)\\
\theta' &= \tilde{N}, && (69)\\
\tilde{N}' &= -3Mu^2. && (70)
\end{aligned}$$

Finally, to match with the solution far away from the boundary layer we require

$$\theta \to -\pi/2, \quad \tilde{u} \to 1, \quad M \to 0, \quad \tilde{N} \to 0 \quad \text{as} \quad \tilde{s} \to -\infty. \qquad (71)$$

We linearise about these to find

$$\begin{aligned}
\tilde{u} &= 1 + De^{\alpha\tilde{s}}, && (72)\\
\theta &= -\frac{\pi}{2} + Ae^{3^{1/3}\tilde{s}} + Be^{(-1/2+i\sqrt{3}/2)3^{1/3}\tilde{s}} + Ce^{(-1/2-i\sqrt{3}/2)3^{1/3}\tilde{s}}. && (73)
\end{aligned}$$

where, to prevent exponentially growing solutions as $\tilde{s} \to -\infty$, we must have $B = 0 = C$. The translational invariance of the problem allows us to set $D = \pm 1$ (depending on whether u_* is greater than, or less than, 1). This leaves us with the shooting conditions

$$\begin{aligned}
\tilde{u} &\sim 1 \pm e^{\alpha\tilde{s}}, && (74)\\
\theta &\sim -\frac{\pi}{2} + Ae^{3^{1/3}\tilde{s}}, && (75)\\
\tilde{N} &\sim A\,3^{1/3}e^{3^{1/3}\tilde{s}}, && (76)\\
M &\sim -\frac{A}{3^{1/3}}e^{3^{1/3}\tilde{s}}, && (77)
\end{aligned}$$

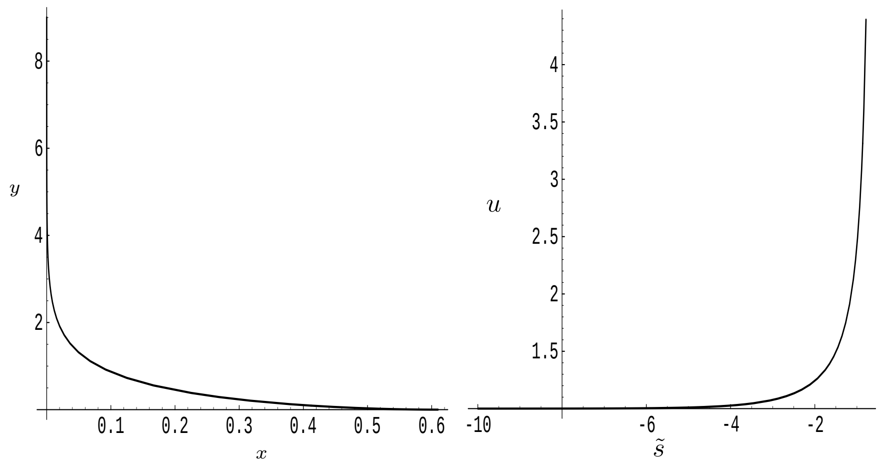

Figure 5: The shape and speed of the fluid in the boundary layer taking $A = 1$, $\alpha = 1$ and $D = 1$ so that $u_* < 1$. This gives an initial velocity of 1.8ms^{-1} and a final x deflection of $X = 1.77 \times 10^{-4}$m.

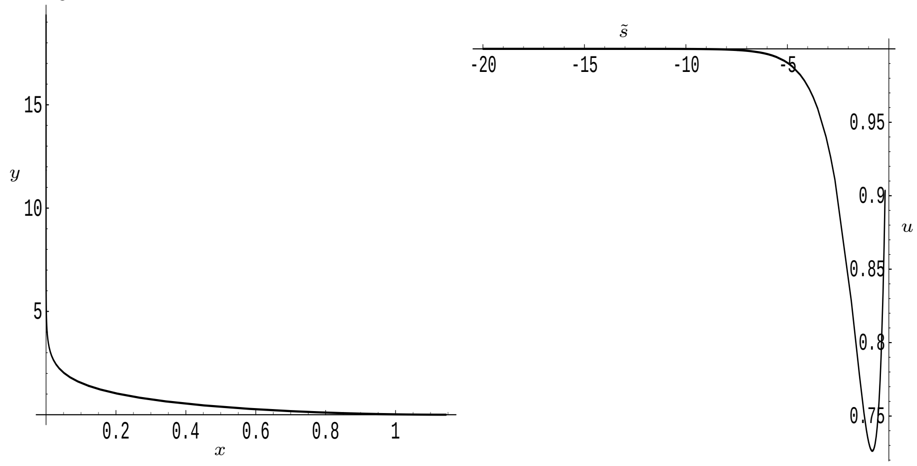

Figure 6: The shape and speed of the velocity in the boundary layer taking $A = 2$, $\alpha = 1$ and $D = -1$ so that $u_* > 1$. This gives an initial velocity of 11ms^{-1} and a final x deflection of $X = 4.3 \times 10^{-3}$m.

as $\tilde{s} \to -\infty$. For each value of A we shoot until we find $\tilde{s}_0$ such that $\theta(\tilde{s}_0) = 0$, and then back out u_* using $\tilde{u}(\tilde{s}_0) = 1/u_*$.

Typical curtain shapes and velocities are shown in Figures 5 and 6 for $u_* < 1$ and $u_* > 1$ respectively. We note that the curtain shapes are qualitatively the same for the two cases, in particular, θ increases monotonically from $-\pi/2$ to 0 in either case. However, when the web speed exceeds the impingement speed (i.e. $u_* < 1$), u is monotonically increasing, whilst for $u_* > 1$, u decreases before increasing to the final web speed.

Since the curtain falls vertically until it reaches a thin boundary layer at the bottom we may approximate the length of the curtain by the height, which is prescribed. Thus, given the impingement speed into the boundary layer u_*, we may back out the dimensional initial velocity $\hat{v}$ using

$$u_* \quad = \quad U\sqrt{2gH + \hat{v}^2}. \tag{78}$$

The deflection of the curtain is given by the value of x at the contact point; when we rescale and return to dimensional variables we find the actual deflection X is given by

$$X \quad = \quad \frac{\left(\mu q^2/\rho\right)^{1/3}}{\sqrt{2gH + \hat{v}^2}} x(\tilde{s}_0). \tag{79}$$

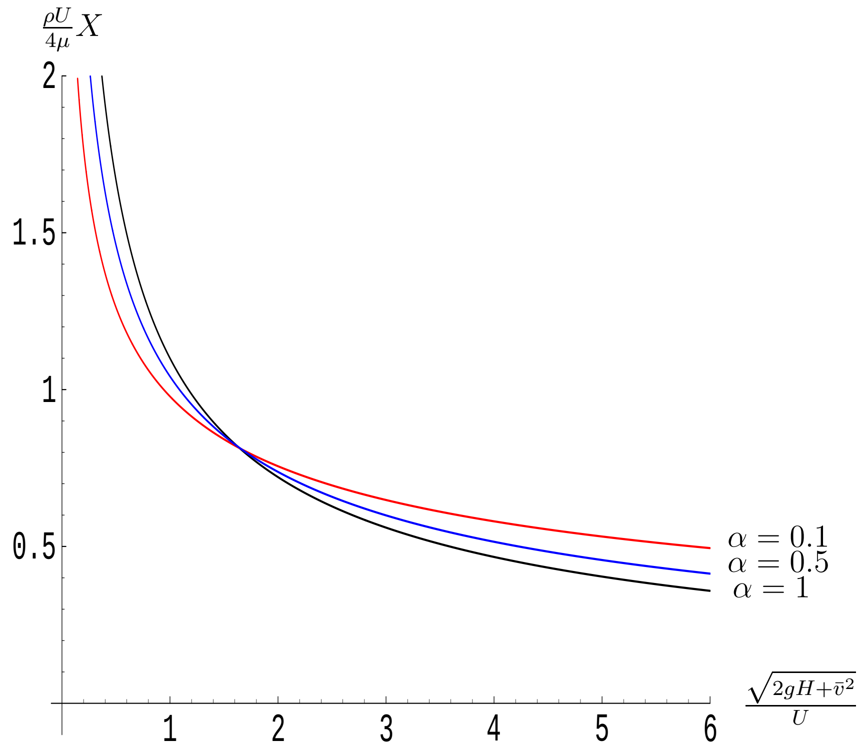

Figure 7: The dimensionless horizontal displacement $\rho UX/4\mu$ versus the impingement-to-web velocity ratio $\sqrt{2gH + \bar{v}^2}/U$.

By varying the shooting parameter A, we may thus plot the solution space for a given value of α. In Figure 7, we plot the ratio of the impingement and web velocities against the rescaled horizontal deflection. This shows that solutions exist for all values of the velocity ratio: perhaps surprisingly there is no significant change in behaviour as the ratio passes through unity. As expected, the deflection increases as the web speed does.

4 Conclusions

In section 2, we derived a basic model for steady-state flow in a liquid curtain, and solved it in the inertia-dominated regime of industrial interest. We found that the model only allows θ to be specified at one end of the curtain. By analysing the characteristics of the unsteady version of the model, one may show that θ must be specified at the top if $T - \rho hu^2 < 0$ or at the bottom if $T - \rho hu^2 > 0$. The former case always holds for inertia dominated curtains, whilst the latter applies to viscous-dominated liquid sheets.

Our simple model therefore does not describe the curtain "turning the corner" to make contact with the moving web. In section 3 we showed the incorporation of a bending moment term leads to a model that allows θ to be specified at both ends of the curtain. The bending moment is negligible except in a boundary layer near the substrate where θ varies rapidly. The lengthscale of this region is of order

$$\delta l = \frac{1}{U}\left(\frac{\mu q^2}{\rho}\right)^{1/3} = \left(\frac{\mu}{\rho q}\right)^{1/3} h_1, \tag{80}$$

where, recall, h_1 is the thickness of the coating. Typical industrial parameters may be used to predict that $\delta l \approx 7 \times 10^{-5}$m. This is smaller than preliminary experimental observations would suggest, since the "turnover" region is visible to the naked eye. We suspect that our assumption of a simple Newtonian rheology is responsible for this disparity. By modifying (42), (43) to incorporate viscoelastic effects we hope to obtain a more realistic lengthscale.

The aspect ratio of the turnover region is given by

$$\frac{h_1}{\delta l} = \left(\frac{\rho q}{\mu}\right)^{1/3} = (\mathrm{Re}^*)^{1/3}\,, \tag{81}$$

where

$$\mathrm{Re}^* = \frac{\rho U h_1}{\mu} = \frac{\rho q}{\mu} \tag{82}$$

is the Reynolds number based upon the curtain thickness (rather than the curtain length l). By applying a quasi-one-dimensional model, we have therefore assumed implicitly that Re^* is small. Otherwise the flow in this region becomes fully two-dimensional; previous numerical studies [1] have shown that a heel forms when Re^* is significantly greater than 1. Since $\mathrm{Re}^* = (4\alpha)^{3/2}$, it is formally inconsistent to treat α as order one while using a one-dimensional approximation. Nevertheless, we expect our boundary layer model to give qualitatively correct results provided reasonably small values of α are used.

We have found that our model admits steady solutions for all values of the ratio u_* between the impingement and web velocities. We are currently investigating the linear stability. One might expect, for example, the curtain to lose stability if the impingement velocity is significantly greater than the web velocity (i.e. $u_* > 1$). We may be able to associate a loss of stability in the turnover region with industrially-observed defects such as air entrainment. Alternatively, it may be necessary to incorporate aerodynamic forces on the curtain, which we also aim to examine in future studies.

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