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Corner selection and pearling transition for drops sliding down a plane.

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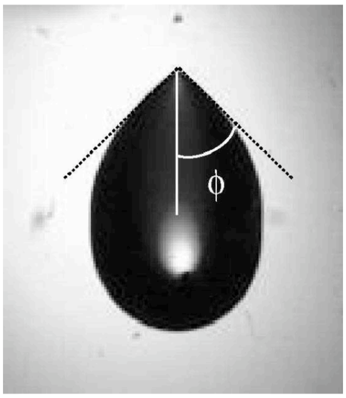
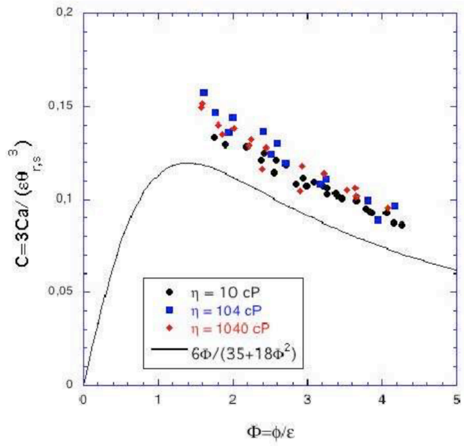
As well known from coating practice, a contact line can not move faster than a critical velocity. Above this critical velocity, a tape pulled out of a bath entrains a liquid film coating the tape. In conditions of partial wetting, at moderate speed, the coating is imperfect, the film remaining confined inside a triangular shape, with a sharp tip at its apex [1]. This sharp "corner" is a typical example of point singularity on interfaces, occuring here in addition on a contact line that breaks into two segments, separated by an angle 2ϕ. In turn, this singular point can become unstable and emit droplets of liquid entrained by the tape (pearling transition). Similar phenomena, perhaps easier to reproduce and to controle (figs a and c), are observed at the rear front of drops sliding down a plane [2,3], when the capillary number $Ca=\eta U/\gamma$ is progressively increased (U drop velocity, η viscosity, γ surface tension).

It has been shown, recently, that both the interface and the flow field near the corner tip can be described as similarity functions of the distance to the tip, the free surface being nearly conical in its vicinity [4,5]. However, up to know, matching this solution with the flow near contact line remains a challenge and the selection law of the ϕ angle remains essentially empirical. It is in general assumed that this angle stays at a marginal value at which both segments of the contact line move on the substrate just at the critical velocity (Blake's law sinϕ~1/Ca), but this has never been proved definitely starting from a rigorous analysis of hydrodynamics. Another problem is also that, without this matching, the similarity solution for the tip exists at any value of the capillary number, which seems to prevent any prediction of the pearling threshold.

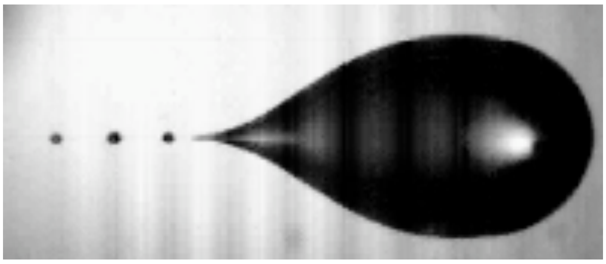
We have investigated, both experimentally and theoretically, these two questions on drops sliding down a plane, in condition of partial wetting [6]. In our model, built in the lubrication approximation (small interface slope, Stokes flows), an imperfect matching can be built between the similarity shape and the flow near the contact line that leads to a selection law for ϕ that is in reasonable agreement with our experiment (fig b). Blake's law is just the low drop speed limit, the prefactor being a bit different than expected. At higher drop speed, a maximal value of Ca is observed that occurs when ϕ becomes close to 30°. No corner can be stable above this critical velocity, but solutions depositing a static rivulet at the rear of the drop can be built from lubrication equations in the limit of slender drops (fig d). The breaking of this rivulet into droplets initiates the pearling transition. We provide laws governing the size of the rivulet, and thus of the deposited drops, above threshold, that are consistent with our observations and measurements.



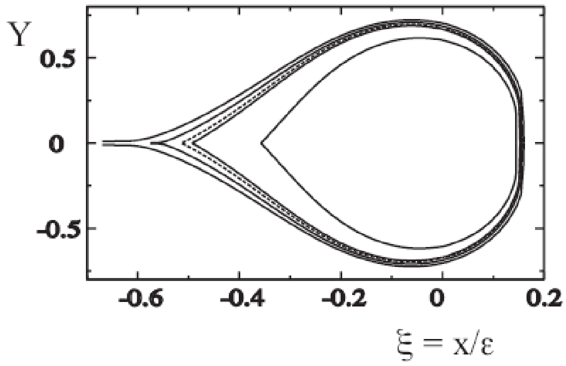
(a)



(b)



(c)



(d)

***Figure**: (a) At high enough velocity a silicon oil drop sliding down a plane coated with fluoropoymers displays a "corner" of opening angle 2ϕ ; (b) angle ϕ for three different oils, conveniently rescaled (see [6]) versus capillary number Ca, and compared to the theoretical curve; (c) a pearling drop, (d) its equivalent calculated in the slender limit from lubrication equations (ε<<1), and a few intermediate shapes.*

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