Thin-film flow on a stationary or uniformly rotating horizontal cylinder subject to a prescribed uniform shear stress at the free surface

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Abstract

We undertake a comprehensive analysis of thin-film flow on a stationary or uniformly rotating horizontal circular cylinder subject to a prescribed uniform shear stress at the free surface of the film.

In the case of a stationary cylinder we describe in detail the properties of the three most relevant kinds of solutions, namely "full-film" solutions, "curtain" solutions and "shock" solutions. In particular, we give an exact (closed-form) expression for the shape of the free surface in each case. We show that the full-film solution always has forwards flow throughout the film, but that both the curtain and shock solutions always have a region of recirculating flow. The subcritical curtain solution also always has a region of backwards flow near the cylinder. For the full-film and shock solutions we calculate the maximum supportable mass, the latter as a function of the location of the shock. We also show how higher-order gravity effects can smooth the shock present in the leading-order shock solution for rimming flow but not for coating flow, whereas higher-order surface-tension effects can do so in both situations. In addition, we use a simpler approach to confirm the stability results obtained by earlier authors.

In the case of a uniformly rotating cylinder there are two different full-film solutions, namely a "Moffatt mode" and a "shear mode". The Moffatt mode is qualitatively the same as the full-film solution on a uniformly rotating cylinder in the absence of a prescribed shear stress, whereas the shear mode is possible only for sufficiently strong shear in the opposite direction to the rotation of the cylinder and is qualitatively similar to (but not identical to) the full-film solution in the case of a stationary cylinder when the prescribed shear stress is in the opposite direction to the rotation of the cylinder. We describe the properties of both modes in detail. In particular, we show that the Moffatt mode always has forwards flow throughout the film, but that the shear mode always has a region of recirculating flow and sometimes has forwards flow on the free surface. In addition, we calculate the maximum (and in the case of the shear mode also the minimum) supportable mass for both modes. Finally, we determine the stability of both modes.