WITHDRAWAL OF A CYLINDER FROM AN ELLIS FLUID

M.Javidi*, M.A. Pope**and A.N.Hrymak
*Department of Chemical and Biochemical Engineering
Western University, London, ON, Canada, N6A 5B9
** Department of Chemical and Biochemical Engineering
Princeton University, Princeton, NJ, USA, 08544

Presented at the 16th International Coating Science and Technology Symposium,
September 9-12, 2012, Midtown Atlanta, GA[1]

Introduction

Dip coating or free-withdrawal coating is a popular and low cost method of depositing a thin, uniform layer of liquid onto a substrate (Middleman, 1977). In the dip-coating process the surface to be coated is initially immersed in the coating fluid and then withdrawn. To better understand the interfacial dynamics of free surface flows and estimating the entrainment of a liquid film on a solid substrate, initial analysis was carried out by Landau and Levich (1942) on Newtonian fluids. The theory was later modified by White and Tallmadge (1967), to include the gravitational force. However, the number of studies on coating thickness prediction of non-Newtonian flows is limited, due to the complexity of the behaviour of non-Newtonian fluids. In this work the Ellis constitutive equation has been applied, due to the small shear rates occurring at the surface of coated films for film thickness prediction required in highly reproducible coatings. Applying Ellis constitutive equation seems reasonable to expect better agreement of film thickness data with a theory based on a constitutive equation that included rheological

---

[1] Unpublished. ISCST shall not be responsible for statements or opinions contained in papers or printed in its publications.

behaviour near zero shear rate as well as intermediate shear rate, due to the deficiency of power law model in reflecting the behaviour of non-Newtonian fluids near zero shear rates.

Theory development and Mathematical Analysis

The theory developed based on breaking the flow into three regions. The flow in each region can be described using the Navier-Stokes equation, solving the flow equations with the usual wall and interface boundary conditions and using the matching criteria to solve for the flow.

Assuming one-dimensional flow, equation 1 can be used to describe the flow in all three regions.

$$\frac{d}{dx}\left(\sigma(C_V^P - C_R^P)\right) - \rho g - \frac{1}{r}\frac{d}{dr}(r\tau_{rz}) = 0 \qquad (1)$$

Ellis fluid is characterized by the following equation:

$$\mu = \frac{\mu_0}{1 + (\tau_{rz}/\tau_{1/2}\ )^{\alpha-1}} \qquad (2)$$

Using the equation (1) and replacing the Ellis constitutive equation in the shear stress, the velocity can be calculated. Matching the flow rate of region 1 and region 2 by considering the conservation of mass, a third order non-linear differential equation has been achieved. This equation solved matching the curvature of second and third regions. The variables of the present work are related to the corresponding variables of Roy and Dutt (1981) and consequently, this equation can be reduced to the Power law and Newtonian fluid.

Comparison with Experimental data and previous models

The experimental results have been extracted from a suspension of 1Vol% high refractive index titanium dioxide particles, dispersed in a matrix of Dymax 1186-MT resin for applications in medical devices. The rheological properties of these suspensions were analyzed and input as parameters into this model, Newtonian and power-law models for free withdrawal coating, explained in the former literature to assess the predicted outcome of various models on the coating thickness. The Newtonian model was obtained by White and Tallmadge (1967) and the Power law model was derived by Roy and Dutt (1981) for withdrawal of a cylinder.

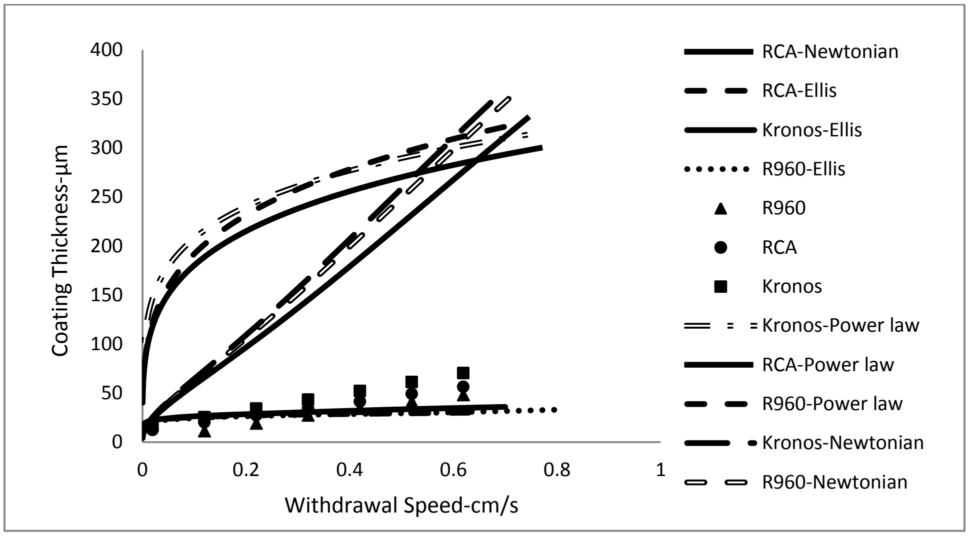

Figure 1: Ellis, power-law and Newtonian models are compared with the experimental data of the film thickness.

Discussion

The Ellis model prediction has better agreement with experimental results in low velocities. Its predictions are in the same order of magnitude of experimental data, but the model fails at higher velocities in these systems. Based on the presence of low shear rates near the surface of coating film (Tallmadge, 1968), better film thickness prediction was expected by the Ellis model, but the complicated behaviour of the system and particle-particle and particle-fluid interaction indicates high shear rates play a major role in the system. These high shear rates occur within the second and third regions, which directly affects the final film thickness in the first region. Ellis model is not capable of covering the behaviour of shear thinning fluids in high shear rates and further deviation occurs between experimental data and predicted results in higher velocities. For

Newtonian and Power-law models, this capability is even more limited for both high and low shear rates and resulting in incorrect prediction of thickness by these models. Therefore, the viscosity in higher shear rates can be investigated by applying the viscosity of second Newtonian plateau of viscosity vs. shear rate curve. Figure 6, illustrates the result of Newtonian model with the viscosity of second Newtonian plateau of the rheological curvature for predicting the final thickness at higher velocities.

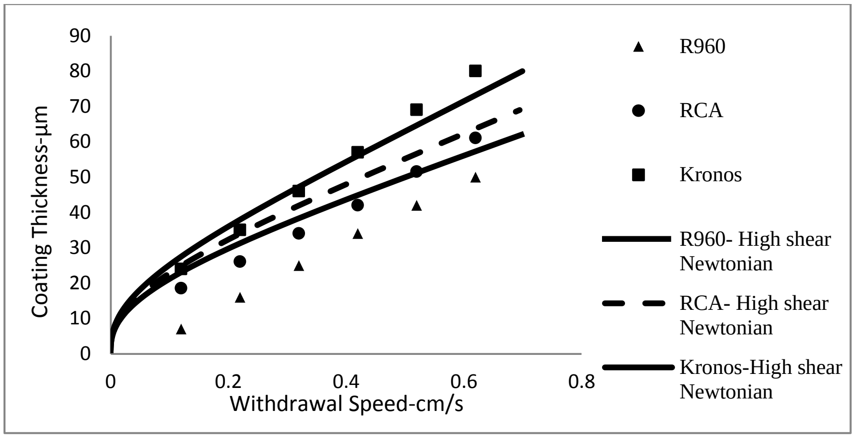

Figure 2: Newtonian prediction of coating thickness using the viscosity of high shear rates in the second Newtonian plateau of the rheological curvature compared to experimental data of three different grade of TiO_2

This indicates significant effect of high shear rates involved in the system and dominant role of shear rates in prediction of the film thickness in these systems.

References

[1] Landau, L., Levich, B., 1942. Dragging of a liquid by a moving plate, Acta physicochemical, U.R.S.S, 17, 42-54.

[2] Middleman, S., 1977. Fundamentals of polymer processing, first ed. McGrawHill, United States, pp. 211-230.

[3] Roy, S.C., 1971. Withdrawal of cylinders from non-Newtonian fluids, Can. J. Chem. Eng. 49(5), 583-589.

[4] Tallmadge, J.A., 1968. The method of addition for deriving continuous withdrawal equation, A.I.CH.E. Journal 14(5), 837-838.

[5] White, D.A., Tallmadge, J.A., 1967. A gravity corrected theory for cylinder withdrawal, A.I.CH.E. Journal 13(4), 745-750.