Locating the jumps in the partially submerged rotating disk problem

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Partially submerged rotating disks have been used for many years as contactors and heat transfer equipment in industries including power, chemical, medical supplies, and sewage, and even as skimmers in oil spills. Even though the film thickness along the disk is a fairly critical factor in determining mass and heat transfer process, it has yet to be completely predicted.

A full numerical solution must include the pool, the disk rotating in the pool and the thin film flowing on the disk on the air side – a fully three dimensional problem with two length scales: the film and the pool – which must have discouraged analysts attempting to solve it. In his MS thesis, Valenzuela (1977) solved the lubrication flow problem for the film on the disk rotating counterclockwise, and determined that the loci of constant film thickness are circular arcs with center on the horizontal line going through the center of rotation, displaced to its right proportionately to the square of the film's thickness. (Afanasiev et al., 2007, and Parmar et al., 2009, identified these same characteristics thirty years later.) When Valenzuela attempted to use the Landau-Levich equation as a boundary condition at the withdrawal side of the pool, he found that the characteristics crossed and concluded that there was no boundary condition at the pool's surface and proposed an empirical solution that consisted of what is described below as fan sections that are patched together in a way that roughly agrees with his observations. Afanasiev et al. successfully applied the Landau-Levich boundary condition on a half-submerged rotating disk, at a rotational rate where the characteristics did not cross.

This presentation shows how the problem is resolved using the Landau-Levich equation at the withdrawal side of the pool, even when the extensions of the characteristics that emanate from the pool cross, by applying the basic concepts of the method of characteristics. It also shows that there is qualitative agreement between Valenzuela's experiments and the present predictions.

The solution of the lubrication film is constructed by Valenzuela, Afanasiev et al. and Parmar et al. in the traditional way: it accounts for the effect of gravity g and viscosity ν in the momentum equation, assumes no-slip at the disk's surface, which rotates at an angular velocity ω, and no-shear at the film's free surface. The surface tension σ is neglected. The solution is then closed using the mass conservation equation. The governing differential equation for film thickness becomes:

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$$-Y\frac{\partial \Delta}{\partial X}+(X-\Delta^2)\frac{\partial \Delta}{\partial Y}=0 \tag{1}$$

where $X \equiv x/R$, $Y \equiv y/R$ and $\Delta \equiv \delta\sqrt{g/(R\omega\nu)}$; and R and δ are the radius of the disk and the film thickness, respectively. The characteristics for this film are lines $Y(X)$ of constant film thickness responding to $dY/dX|_{\Delta} = -(\partial\Delta/\partial X)/(\partial\Delta/\partial Y)$. Combining this with eq.(1) and integrating yields the equation for the characteristic of thickness Δ.

$$Y^2+\left(X-\Delta^2\right)^2 = Y_o^2+\left(X_o-\Delta^2\right)^2 \tag{2}$$

where (X_o, Y_o) is a point on the characteristic. Note that eq.(2) is the equation of a circle with center at the coordinate $(\Delta^2, 0)$. Along the pool surface where the film is being withdrawn film thickness is provided by the Landau-Levich equation, which in dimensionless terms is

$$\Delta = 0.944 Ca^{1/6} X_0^{2/3} \tag{3}$$

where $Ca \equiv \rho\,\nu\, R\omega/\sigma$, X_0 is the dimensionless abscissa of the characteristic emerging from the pool, and ρ is liquid density. As pointed out by Valenzuela, when eq. (3) is applied the arcs cross; but this actually is an indication that there is a jump in the solution. (Afanasiev et al. seemed to have chosen a condition that is so slow that the characteristics do not cross.) Jumps tend to start where two different sections of the solution meet or where two adjacent characteristics cross. The latter are precursors to a jump which could appear in the section that emerges from the pool. Assuming the pool surface to be located at the ordinate Y_0, a precursor for each characteristic originating at (X_0, Y_0) is found by differentiating eq. (2) by X_0 and finding (X, Y)

$$X = X_0 - \frac{(X_0-\Delta^2)}{2\Delta\partial\Delta/\partial X_0} \tag{4}$$

where $\partial\Delta/\partial X_0$ is found from eq. (3) and Y is found using eq. (2). At a jump the continuous solution breaks down. Invoking mass conservation across the jump defines the slope of the jump at that point. Thus, where two characteristics of thicknesses Δ_1 and Δ_2 meet, the slope of the jump Y_J is

$$Y_J\frac{dY_J}{dX} = -X + \frac{\Delta_1^2+\Delta_1\Delta_2+\Delta_2^2}{3} \tag{5}$$

Jumps terminate on the abscissa, $Y_J = 0$, which originates a section where the characteristics fan out of the termination point, with film thicknesses varying continuously between the two thicknesses across the jump at that point. If the termination point is at $(X_2, 0)$, the film thickness at any point (X, Y) in the fan section is given by eq (2):

$$\Delta = \left[\frac{Y^2+X^2-X_2^2}{2(X_2-X)}\right]^{1/2} \tag{6}$$

Given these relations, it is now possible to construct a solution for different capillary numbers Ca and locations of the pool's surface Y_0. Figure 1 is a typical solution for a rotating disk with $Ca = 0.09$ and $Y_0 = -0.5$. Section 1 emerges from the pool obeying eqs. (2-3). The liquid film runs off of the rim of the disk in the fourth quadrant of the disk. As the section enters the first quadrant, the characteristics move away from the rim where a fan Section 2 originating at $(1,0)$ starts. The film thicknesses in Section 2 range from the thickness of the adjacent characteristic of Section 1 that passes through $(1,0)$ and zero thickness at the rim of the disk. (As eq.2 shows, all characteristics that are concentric with the disk must have zero thickness, i.e., $\Delta = 0$.)

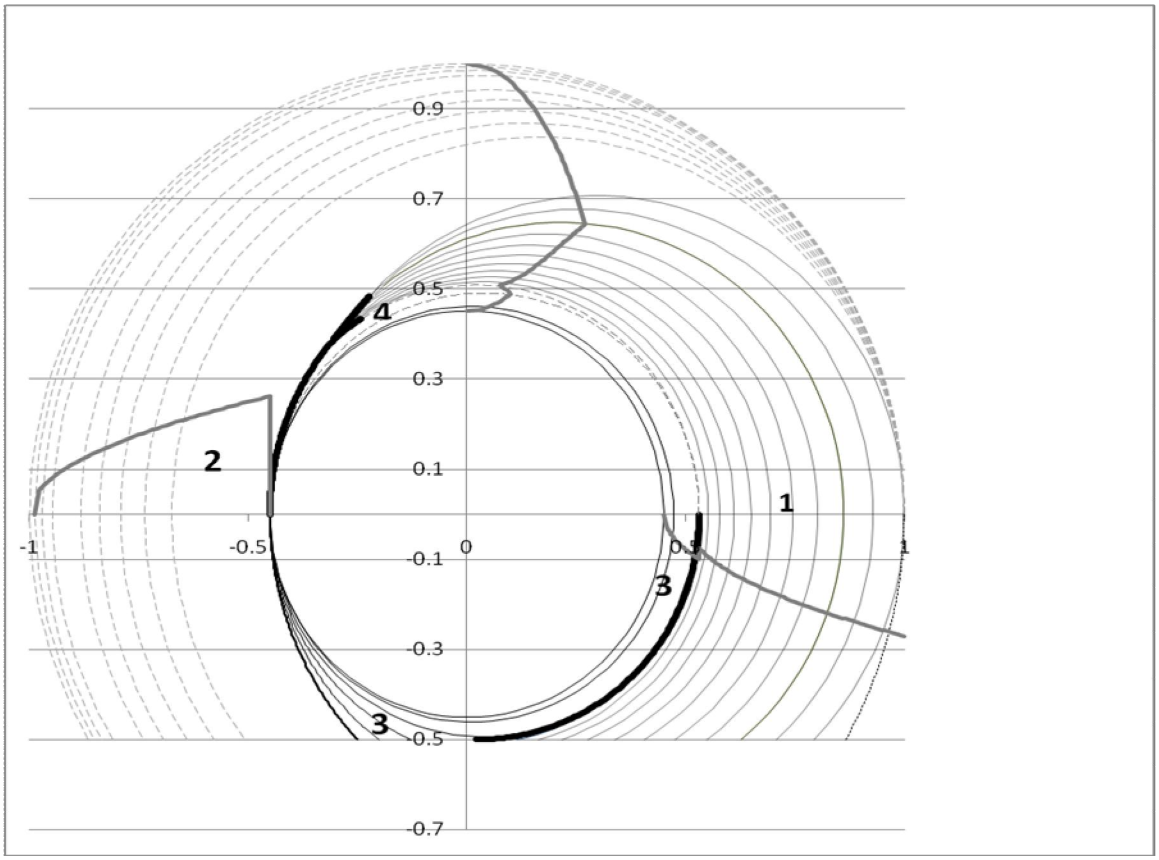

Figure 1

As can be seen, the characteristics of a fan section never cross each other. A jump, however, originates on the border between Sections 1 and 2 when adjacent characteristics in Section 1 at its outer edge start crossing each other. The location of the origination point is given by eqs. (2) and (4) and the jump is established by integrating its slope using eq.(5). The jump between Sections 1 and 2 reaches the abscissa by separating other flow sections which are described later. This jump terminates on the abscissa, and from there a fan Section 3 originates. The distance between the disk's center and the origination point is slightly less than the distance between the center and the pool. The thickness of the film in Section 3 ranges from the thickness of the characteristic of Section 2 going through the originating point of Section 3 and zero. Therefore, the inner circle is a characteristic of zero film thickness which is concentric with the disk.

Once Section 3 reaches the pool's surface, where liquid is being withdrawn, a jump needs to form between Sections 1 and 3. It originates where the slope of the nascent jump at the pool's surface turns positive, i.e., the point on $Y = Y_0$ where $dY_J / dX = 0$ in eq. (5) and the film thicknesses across the jump are given by eqs. (3) and (6). The jump's trajectory is then defined by integrating eq. (5). It terminates on the abscissa and fan Section 4 originates there, separating Sections 1 and 3. A very short jump is generated between Sections 1 and 4 at a point where adjacent characteristics of Section 1 on the boundary cross, which is defined using eq. (4). The jump intersects the jump between Sections 1 and 2, and continues as a jump between Sections 2 and 4 found, again, by integrating eq. (5).

The presentation will show predictions for characteristics of several of the nine different conditions with Ca and Y_0 varying from 0.045 to 0.135 and from -.7 to -.3, respectively. Comparisons will be made with Valenzuela's film thickness measurements and eq. (3), and his measurements of the height above the pool over which it wets.

Although capillarity has been introduced at the pool's free surface at the line of withdrawal, the model neglects its influence at the jumps and at the inner and outer contact lines. One can speculate that the capillary effect at the jumps merely rebalance forces, just like viscosity influences momentum and energy balances in shocks in compressible flow, and therefore should have a very localized effect. With the exception of the third quadrant's outer rim, the calculation predicts a contact angle in the edge of the film to be everywhere at 90^{o}; which is probably too high as the liquid is an oil and the disk was made out of galvanized steel. This would probably increase the rise of the film's rim above the pool's surface even further than predicted, but this opposes the trends observed in the experiment. Centrifugal force, although seemingly small, is applied along the entire film and most probably explains the discrepancy between measurements and prediction.

This presentation's contribution is the prediction of film thickness throughout the entire partially submerged rotating disk for lubricating flow that accounts successfully for the Landau-Levich boundary condition at the withdrawal side of the pool's surface, and the use of the method of characteristics to identify the jumps in film thickness.

References

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