Asymptotic structure of a dewetting thin liquid film

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In recent years the dewetting behavior of a thin liquid film on a solid substrate has received much attention. Such behavior is important for the preparation of polymer films, for microfluidic devices, and other applications.

When a thin liquid film dewets, it may form a rim which spreads outwards, leaving behind a growing dry region. Experiments by Reiter and others [1] involving dewetting films of polystyrene on polydimethylsiloxane-coated silicon reveal an asymmetric rim shape, and spreading at various rates. To explain these observations, different mechanisms have been proposed, including slip at the liquid-solid interface [2].

We make use of a strong-slip model, which has been derived previously [3]. The coating thickness may vary with position x and time t, and is denoted by $h(x, t)$, while its velocity is $u(x, t)$. The coating evolution is governed by the Navier-Stokes equations for momentum and mass conservation of a viscous incompressible liquid, and the stress-strain relationship is assumed to be Newtonian. At the impermeable substrate, the Navier slip condition

$$u = B\frac{\partial u}{\partial y}$$

is imposed while the normal component is set to zero. Here the quantity B is a slip length, with $B = 0$ representing the no-slip case, while B becoming infinite represents the limit of perfect slip. At the free surface normal stresses arise from capillarity, and there are assumed to be no tangential stresses.

The model is based on the lubrication approximation. Neglecting inertia terms, we arrive at the strong-slip model proposed by Münch et al. [3] and Karagupta et al. [4]:

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[1] Unpublished. ISCST shall not be responsible for statements or opinions contained in papers or printed in its publications.

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0\,, \tag{1a}$$

$$4\frac{\partial}{\partial x}\left(h\frac{\partial u}{\partial x}\right) + h\frac{\partial^3 h}{\partial x^3} = \varepsilon u\,. \tag{1b}$$

In dewetting, we suppose that the film free surface meets the substrate at a contact line at $x=s(t)$, where $s(t)$ is to be determined. Appropriate boundary conditions at the contact line are

$$h = 0, \quad \frac{\partial h}{\partial x} = \lambda, \quad h\frac{\partial u}{\partial x} = 0, \qquad \text{and} \quad u = \dot{s}(t). \tag{2}$$

so the contact angle is arctan λ. In the undisturbed region we impose

$$h \to 1, \quad u \to 0 \quad \text{as } x \to \infty. \tag{3}$$

As initial data we set

$$s(0) = 0 \quad \text{and} \quad h(x,0) = h_{\text{init}}(x) \quad \text{for } x \geq 0, \tag{4}$$

where $h_{\text{init}}(x)$ is smooth, positive, and satisfies the boundary conditions.

In the limit ε goes to zero, with t being order 1, the asymptotic structure consists of an inner region, close to the contact line, and an outer region. (See Figure 1.) The effects of capillarity are confined to the inner region. We describe the large-time behavior of these regions. For the inner region at large time we find a similarity solution describes the growing inner rim, and compute this solution numerically. The outer region is somewhat complicated, and at large times we find that it can be thought of as three subregions. In an outermost subregion (O3) a traveling wave ansatz describes the film. Closest to the contact line, we find a time-dependent solution in a frame moving with the contact line speed (O1). Finally, between these two inner subregions there lies one (O2) in which a WKBJ ansatz applies.

We used time-dependent numerical simulations for (1), and for the related outer region problem. In Figure 2 we compare the solution at t=5 to the inner and outer solutions. We also show a composite solution $h^{(c)}(x)$. Provided t is not too large, this is a good approximation to the solutions of (1). To see the separation of the outer solution described above takes quite long times. However, in long-time simulations of the outer problem this separation is visible; Figure 3 compares the outer region solution to those for subregions O1 and O3.

At larger times, further changes in the structure of the rim occur.

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References

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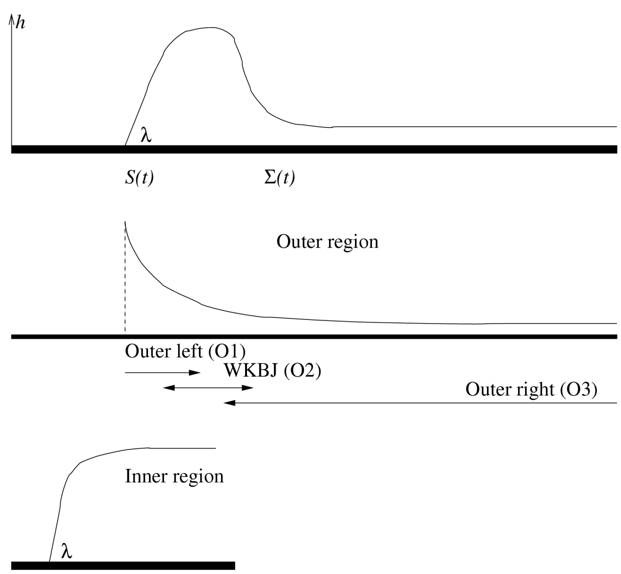

Figure 1: The spreading rim (top) is described as an inner region, and an outer region. The latter is divided into three sub-regions (bottom).

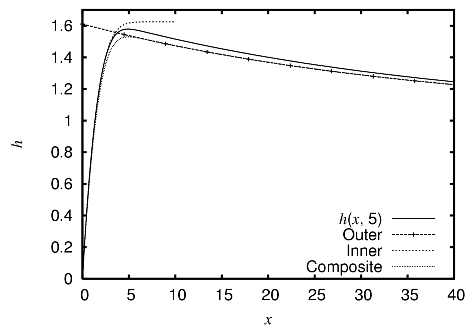

Figure 2: Solution to (1) at t=5 for ε=0.002. This is compared to the inner and outer solutions, and a composite expansion.

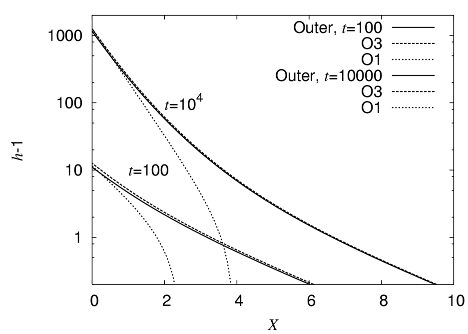

Figure 3: Solution to the outer problem at t=100 and t=10000, compared to the O1 outer left and O3 outer right solutions.