Spreading and infiltration of droplets on permeable layered substrates

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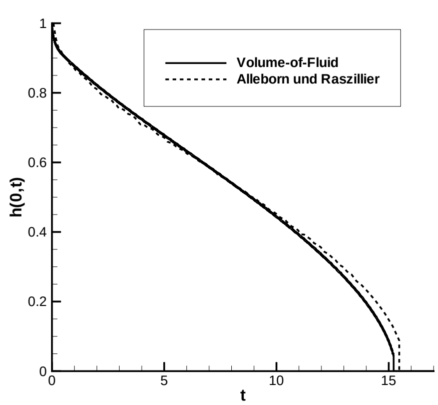
Spreading and infiltration of liquid on permeable substrates is an important process in liquid film coating and printing. Applications cover leveling and absorption of coating colors in paper coating and conventional ink jet printing. The absorption kinetics of the liquid droplet applied on the porous substrate determine the speed of further substrate processing. The fluid flow in the porous substrate due to capillary action, which may continue even after complete absorption of the droplet, has a significant influence on the final quality of the coated product. For an overview over available literature see e.g. [1, 2, 3].

In this paper, the spreading and infiltration dynamics of droplets on unsaturated, possibly layered porous substrates are investigated numerically. Based on previous studies [1, 2, 3], in which the flow of flat films and droplets (small contact angles) on the substrate surface has been investigated in the framework of the lubrication approximation, the present work complements this work towards larger contact angles ($> 45°$) and the droplet impingement dynamics. To do so, the spreading of the droplet above the substrate has been captured with an adapted and extended version of an axisymmetric volume-of-fluid method, see e.g. [4, 5]. The fluid flow within the porous substrate, caused by the droplet penetrating into it, is treated as unsaturated flow in the framework of Richards' equation

$$\frac{\partial c}{\partial t} = \nabla \cdot \left(\frac{\mathbf{k}}{\mu} \nabla p_p \right) \tag{1}$$

and was coupled to the fluid flow above the substrate surface. Here, c denotes the local volumetric fluid content of the porous substrate, t the time, $\mathbf{k}$ the saturation dependent permeability tensor, μ the liquid viscosity and p_p the pressure inside the pores of the substrate. This approach allows to describe fluid transport in the porous layer after complete absorption of the liquid above the substrate surface and extends thus conventional wetting front models. The fluid content c is related to the pressure in the porous material, modeled here by the van Genuchten-Mualem model (sorption curve), with the model parameters $m = 5$, $n = 10$ and $\alpha = 7.12 \times 10^{-9}$ Pa^{-1} for characterizing the sorption curve [3]. A detailed description of the basic equations governing the flow in the porous material and the coupling conditions can be found in [3]. Following the nondimensionalization as used in [1, 2, 3], the droplet profile $h(r,t)$ has been scaled with h_e, the vertical and horizontal coordinates, r and h, respectively, have been scaled with R_e, whereby h_e and R_e denote the equilibrium height and radius of a static droplet with contact angle θ_e. The spreading process is characterized by the Bond number ($Bo = \rho g R_e^2/\sigma$) and the infiltration into the substrate by the permeability number ($Pm = 3k_z R_e^2/h_e^4$ with the vertical permeability k_z) and the suction number ($Su = p_c R_e^2/\sigma h_e$ with the capillary pressure p_c). The time scale is $T = 3\mu R_e^4/\sigma h_e^3$. For the simulations presented here, a droplet ($\mu = 40$ mPa s, $\rho = 1000$ kg/m^3, $\sigma = 30$ mN/m) of size 100 pl has been deposed on the substrate surface, whereby the apparent contact angle of the liquid-solid system is assumed to be $\theta_e = 45°$. The porous layers have isotropic permeabilities ($k_r = k_z$) entering the permeability tensor $\mathbf{k}$ and a constant porosity $\phi = 0.25$. The results presented in the following are provided in non-dimensional quantities.

The developed numerical algorithm has been validated first through the simulation of a sessile droplet penetrating into a single porous substrate using the present method and a code based on the lubrication approximation for the dynamics of the droplet [3]. In Fig. 1 some results are provided showing a comparison of the droplet height $h(0,t)$ (left) and the droplet profiles and contour plots of the fluid content $c(r,z,t)$ at two time instants (right). In the right figures, the dark droplet profile and the black contour lines represent the present method, whereas the data based on [3] are shown by a lighter droplet profile and the grey-scale contour plot. A very good agreement throughout the entire penetration process has been found.



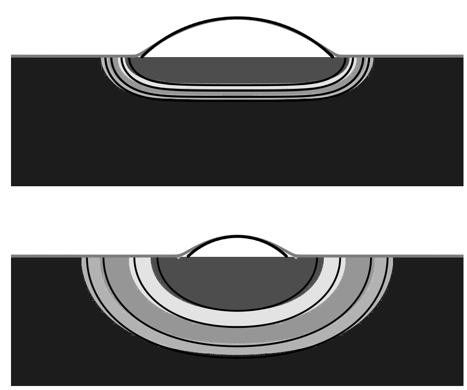


Figure 1: Comparison of the present numerical method with the lubrication model of [3]: (left) height of the droplet $h(0,t)$ and (right) two time instants of spreading and penetration (present method: black droplet profile and black contour lines).

For illustration of the present method, the influence of the layer thickness ratio d_1/d_2 of a two-layer substrate system has been investigated, where the indices 1 and 2 denote the upper and lower layer, respectively. For this study, the total thickness $d_1 + d_2 = 1.14$ and the permeability ratio $Pm_1/Pm_2 = 0.05$ ($Pm_1 = 1 \times 10^{-7}$) are taken constant. The simulations have been started with a droplet shape of a half circle, i.e. with an initial apparent contact angle of 90°. In Fig. 2 the

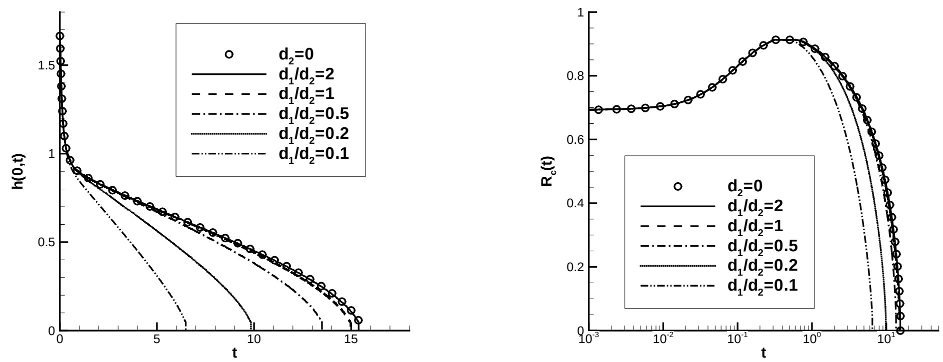


Figure 2: Influence of the ratio d_1/d_2 on the droplet height $h(0,t)$ and the contact radius $R_c(t)$ ($Pm_1/Pm_2 = 0.05$).

droplet height $h(0,t)$ and the contact radius $R_c(t)$ for different layer thickness ratios d_1/d_2 show, that the total infiltration time τ decreases significantly with increasing thickness of the lower, more permeable layer. For a thin upper layer ($d_1/d_2 = 0.1$), τ is 2.4 times smaller than in the case $d_2 = 0$ (single layer of permeability Pm_1). However, the effect of a significantly reduced infiltration time caused by an additional lower layer is noticeable only for $d_1/d_2 < 1$ under the conditions used here. In Fig. 3, contour plots of the fluid content c for $d_1/d_2 = 1$ at three time instants (a-c) and for

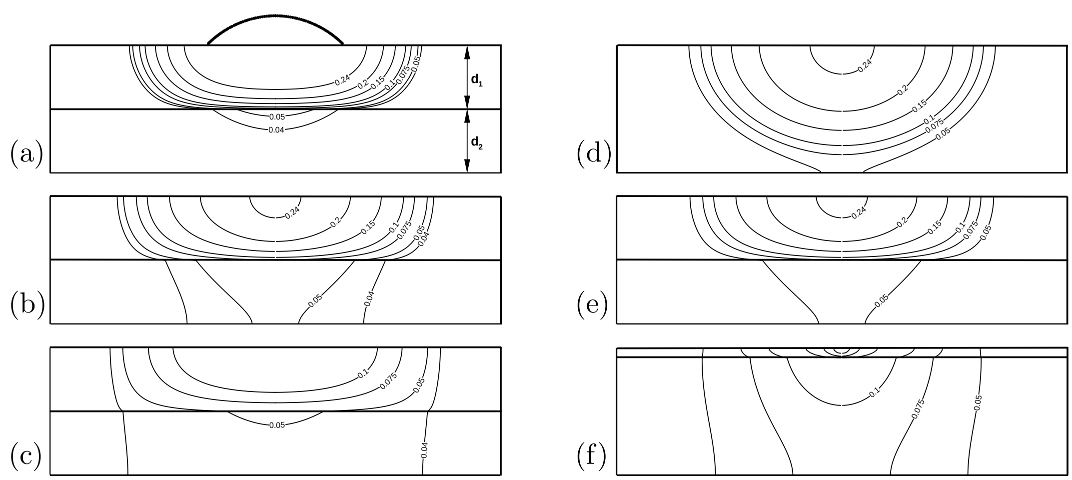


Figure 3: Snapshots of the droplet profile $h(r,t)$ and contour lines of the fluid content $c(r,z,t)$: (a)-(c) three time instants ($t = 6.8, t = \tau = 15.0$ and $t = 25.4$) of spreading and infiltration with $d_1/d_2 = 1$; (d)-(f) influence of the layer thickness ratio d_1/d_2 on the fluid content distribution at complete infiltration $t = \tau$ for the cases $d_2 = 0$, $d_1/d_2 = 1$ and 0.1.

different thickness ratios d_1/d_2 at $t = \tau$ (d-f) give an impression of the flow in the porous layers. The gradients of the fluid content in the upper layer can be seen to be much higher than in the lower one, which is caused be the lower permeability of the upper layer. Different to conventional wetting front models, the Richards' equation used here allows to study the flow in the substrate also for $t > \tau$ as shown in Fig. 3 (c). In the case of a single layer of permeability Pm_1 (d), the infiltration until $t = \tau$ is essentially unaffected by the impermeable boundaries of the substrate. Although the contour lines for $d_1/d_2 = 1$ (e) differ strongly from the single layer case (d), the total absorption time differs only slightly, as was shown in Fig. 2. In the case $d_1/d_2 = 0.1$ (f), the enhancement of the vertical penetration of the fluid through the upper layer due to the higher permeability of the lower layer leads to a much shorter infiltration time.

For illustration of the impingement process under the present conditions, Fig. 4 shows some droplet profiles of a spherical droplet impinging on the substrate with zero velocity. It can be seen, that the initial spreading is very fast, e.g. the $t = 0.06$ profile corresponds to a spreading time period of about 0.2 msec.

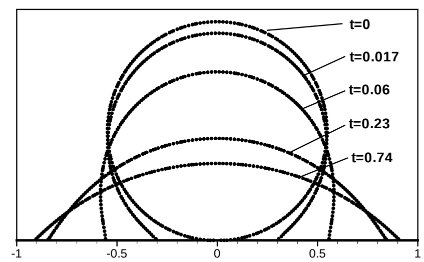


Figure 4: Droplet impingement.

Results of a numerical investigation on the spreading and absorption of droplets with large contact angles on layered porous substrates have been presented. The method has been successfully validated for the case of a flat droplet with a numerical code based on the lubrication approximation [3]. The support of the Deutsche Forschungsgemeinschaft (DFG) is gratefully acknowledged.

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