Density-driven pencil-line streaks in slide flows

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An effective method of coating several different layers on a support is to apply them simultaneously. One of the major sources of non-uniformities encountered in this type of coating process is streaks, broad or narrow lines that are aligned with the direction of coating. This presentation deals with streaks formed on a slide induced by adverse density ordering in the film, that is, when layers of higher density flow over layers of lower density.

There have been a number of presentations over the years on the subject of streak formation on slides. Hayes and O'Brien (1999) and Gaskell et al. (2002) studied the effect an irregularity in the wall of the slide had on the shape of the free surface; while Joos and Devine (1997, 2003, 2004a and b) have predicted and verified the formation of streaks in films of multiple layers with varying viscosity. The latter studies concluded that in films of layers of equal density, a streak would eventually stop developing. In a US patent, Bhave et al. (2002) discuss the catastrophic development of streaks on a slide with multiple layers in a process they called "strike-through", and devised methods to avoid this phenomenon, which originated with more dense layers flowing over less dense layers.

This paper deals with the prediction and verification of exponential growth or decay of streaks in layers cascaded on a slide when their densities are mismatched. Figure 1 shows two layers of liquid flowing down a slide with a 15^{o} inclination from the horizontal. The bottom and top layer flow rate per unit width is, respectively, 0.15 and 0.6 cm^2/s, density is 1.03 and 1.17 g/cc, and viscosity is 30 cP in both layers. To trigger the instability, pillbox shaped disks of different sizes are mounted near the upstream end of the image. The instability is noted by the growth of the streaks in the bottom layer, which contains carbon black to make the streaks visible. Two observations are made plain by the image:

- the streaks grow exponentially; and
- the wavelength of the streak is selected by the film, and is independent of the size of the obstruction.

Model development

As in previous models developed by Joos and Devine, the layers are assumed to be miscible but there is no mass transferred between the layers. This enables modeling the layers with known and uniform viscosities

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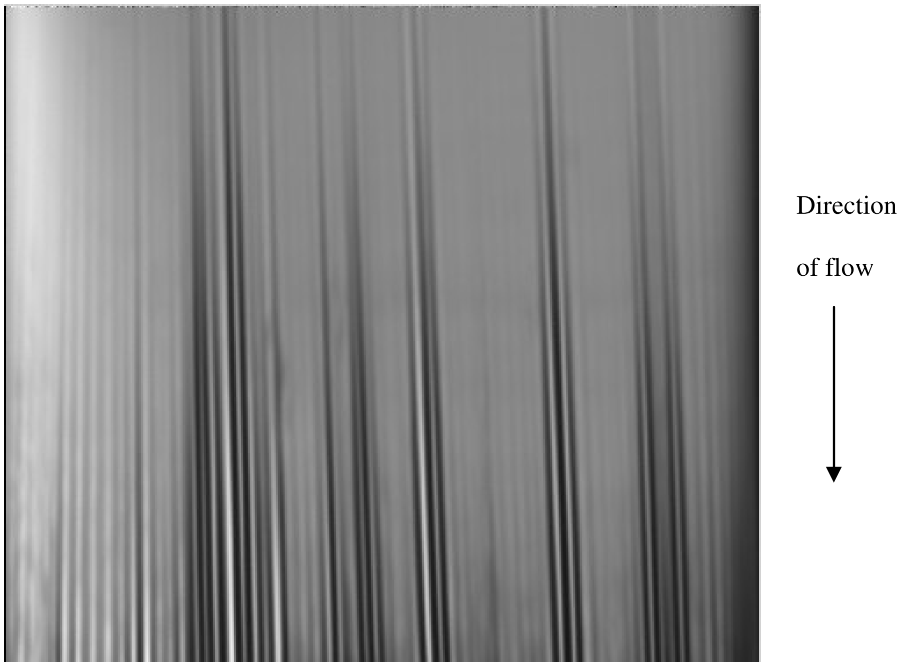

Figure 1 Streak formed in a film of two layers with adverse density placement. The image covers a breadth of about three inches of the slide flow.

and densities, and surface tension to exist only at the interface between the top layer and the air. Furthermore, inertia is neglected. For each layer, the mass and momentum conservation equations are reduced to:

$$\nabla \cdot \vec{V}_i = 0 \quad \text{and} \quad \nabla p_i = \rho_i \vec{g} + \mu_i \nabla^2 \vec{V}_i \qquad \text{(1a,b)}$$

where $\vec{V}_i$ is the velocity vector for the liquid in the i^{th} layer, $\vec{g}$ is the gravitational vector, p_i is the pressure in the layer, and ρ_i and μ_i are its density and viscosity. The y-axis is perpendicular to the slide, and the x-axis is in the direction of the general flow; as a result the z-component of $\vec{g}$ is null.

As opposed to the previous models presented by Joos and Devine, the long wave approximation that leads to the typical lubrication formulations is not invoked. As Figure 1 shows, the wavelength of the streaks is about 2 mm, which is comparable to the film's thickness; therefore all the viscous terms are kept.

An insoluble surfactant is adhered to the free surface. The equation of conservation of surfactant is:

$$\nabla_s \cdot \left(\gamma \vec{V}_s\right) = 0 \qquad (2)$$

where γ is the surfactant surface concentration, $\vec{V}_s$ is the velocity vector at the free surface and $\nabla_s \equiv \{\partial / \partial x, 0, \partial / \partial z\}$ is the del operator without the second component. The interfaces between the layers and at the free surface obey the kinematic condition:

$$v_i = \left(\vec{V}_i \cdot \nabla_s\right)\xi_i \quad \text{at} \quad y = y_i \qquad (3)$$

where v_i, y_i and ξ_i represent the velocity component normal to the slide, the distance of the upper interface of the i^{th} layer to the slide floor for the uniform flow with no streaks, and the normal displacement of the same interface when the flow has been perturbed.

Boundary conditions for the film are no-slip and no-flow through the slide's wall:

$$\vec{V} = 0 \text{ at } y = 0 \;; \tag{4}$$

and, at the free surface, that shear stress at the free surface is balanced by the surface tension gradients induced by surfactant concentration gradients; and that pressure at the free surface is balanced by capillary and gravitational pressure:

$$\tau = \rho_n g_x \xi_n \vec{i}_x + e\nabla_s\left(\gamma/\gamma_o\right) \text{ where } \tau \equiv \mu_{\mathrm{n}}\left\{\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right), 0, \left(\frac{\partial v_y}{\partial z}+\frac{\partial v_z}{\partial y}\right)\right\} \text{at y = y}_{\mathrm{n}} \tag{5}$$

$$p_n + \frac{\sigma}{R} = 2\mu_n \frac{\partial v_x}{\partial x} - \rho_n g_y\,. \tag{6}$$

Here γ_o is the surfactant surface concentration in the undisturbed flow; $e \equiv \gamma_o \partial\sigma/\partial\gamma$ is the surface elasticity, which is assumed constant; R is the free surface's radius of curvature, g_x and g_y are components of the gravitational vector; and $\vec{i}_x$ is the unit normal vector in the x-direction.

Speed, pressure and shear must be matched at the interfaces between the layers:

$$\vec{V}_i = \vec{V}_{i+1} \text{ at y = y}_{\mathrm{i}} + \xi_i \tag{7}$$

$$\tau_i - \rho_i g_x \xi_i \vec{i}_x = \tau_{i+1} - \rho_{i+1} g_x \xi_i \vec{i}_x \text{ at } y = y_i \tag{8}$$

$$-p_i - \rho_i g_y \xi_i + 2\mu_i \frac{\partial v_i}{\partial y} = -p_{i+1} - \rho_{i+1} g_y \xi_i + 2\mu_{i+1}\frac{\partial v_{i+1}}{\partial y} \text{ at } y = y_i \tag{9}$$

Equations 1-9 are solved in the traditional way, using a perturbation method. First a base or zeroth order solution is found, for the unperturbed ($\xi_i = 0,\ \ 1 \le i \le n$) fully developed (i.e., all $\partial/\partial x$ terms negligible) flow. Then the above equations are greatly simplified. As it is typical that the flow per unit width q_i in each layer is pre-determined, it is convenient to include this in the solution to the base flow and find the heights of the interfaces between the layers as part of the solution:

$$q_{0i} = \int_{y_{i-1}}^{y_i} u_{0i}\,dy \tag{10}$$

The base solution is determined from the following set of n nonlinear algebraic equations, that must be solved for $y_i, i = 1,..,n$ where $\mathrm{y}_{\mathrm{i-1}} \le y_i$:

$$q_{0i} = g_x \left\{ \int_{y_{i-1}}^{y_i} \left[\int_0^y \left(\frac{1}{\mu(y')} \int_{y'}^{y_n} \rho(y'')dy'' \right) dy' \right] dy \right\} \text{ where } \rho, \mu(y) = \rho_j, \mu_{\mathrm{j}} \text{ when } y_{j-1} \le y \le y_j \qquad (11)$$

Next, the first order perturbation has to be solved. The field equations (1) are those for creeping flow:

$$\nabla \cdot \vec{V}_{1i} = 0 \quad \text{and} \quad \nabla p_{1i} = \mu_i \nabla^2 \vec{V}_{1i} \qquad (12a,b)$$

Thus, each velocity component satisfies the biharmonic equation. Assuming that all dependent variables have the form $f(y)\exp(rx + jsy)$ (where j is the imaginary unit), each velocity component for a layer has the form: $Ae^{\kappa y} + Be^{-\kappa y} + Cye^{\kappa y} + Dye^{-\kappa y}$ where $\kappa^2 = s^2 - r^2$.

Applying the field equations, boundary conditions and interfacial matching conditions, a linear set of homogeneous equations is formed to solve for coefficients of the type shown in equation (13). By specifying a wavenumber s and requiring that the sum of the squares of the coefficients be equal to unity, one can determine growth rates r for which there are solutions that are not null. This is a non-linear problem, so finding these solutions is not trivial. The technique used to find the growth rates will be discussed during the presentation.

Figure 2 shows an example of the growth predicted for a film of four layers where the density increases with separation from the slide. It appears that there are as many growing modes as number of adversely placed layers, and that the wavelength of greatest instability is comparable to thickness of the layer. For long wavelengths to the right of the peak, growth rates are consistent with predictions using the long wavelength approximation, where it is found that growth rates consistently decrease as the wavelength increases. To the left of the peak, we find ourselves in a regime where viscous dissipation along the width of the streak dampens the growth.

In the presentation we will show a comparison of streaks in two films with the same layer flow rates and viscosities, but one film with adverse placement of density while the other has all layer densities equal.

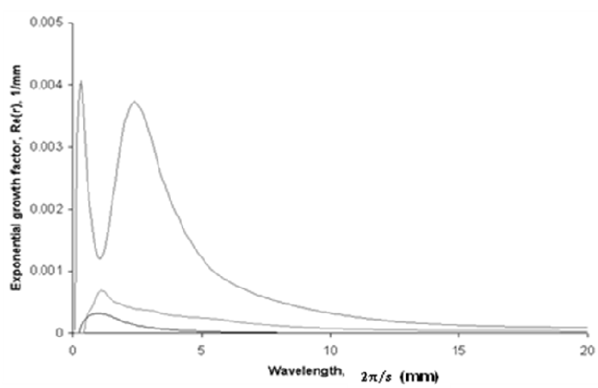

Figure 2 Predicted growth rates for a film of four layers with their densities adversely placed.