

# Dynamical Analysis of Gravity-driven Thin Film over Substrates with Topography

**Zhaomiao LIU<sup>a</sup>, Mark C. T. Wilson<sup>b</sup>, Philip H Gaskell<sup>b</sup> & Harvey M Thompson<sup>b</sup>**

<sup>a</sup> School of Mechanical Engineering & Applied Electronics Technology  
Beijing University of Technology, Beijing 100022, P. R. China

<sup>b</sup> School of Mechanical Engineering  
The University of Leeds, Leeds LS2 9JT, United Kingdom

Presented at the 13<sup>th</sup> International Coating Science and Technology Symposium, September 10-13, 2006,  
Denver, Colorado<sup>1</sup>

Thin liquid film flows over substrates with topography play a key role in many coating processes. The features of such coated films will be influenced by the underlying substrate profile, and this non-uniformity may lead to variations in coating thickness or even instabilities. The present paper focuses on eddy generation, mass transfer and mixing of gravity-driven flow over substrates with wavy and triangular profiles, whose geometry and notations are shown in Fig.1 and Fig.2.

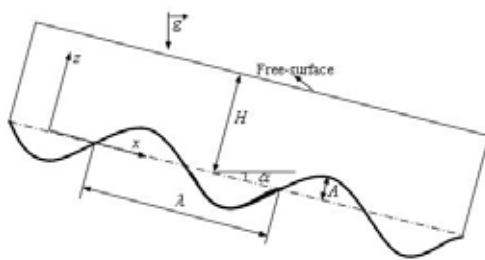


Fig.1 Geometry and notations of the flow over wavy substrate

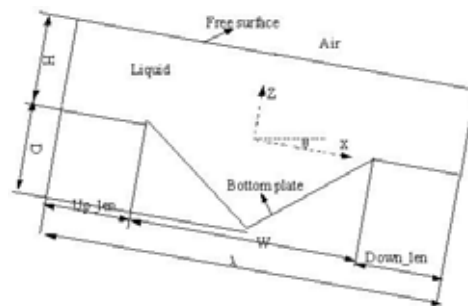


Fig.2 Geometry and notations of the flow over triangular substrate

<sup>1</sup> Unpublished. ISCST shall not be responsible for statements or opinions contained in papers or printed in its publications.

Although a few such analyses of relevance to coating flows have been reported <sup>[1-4]</sup>, in this paper the mechanisms of mass transfer and mixing from eddies are analyzed by making use of computational techniques. The streamlines of the flow are obtained using a Galerkin Finite Element Method. Based on the calculations, the formation and evolution of vortices in the flow are visualized. The influence of the plate topography and shear-thinning coefficient on the film flow is investigated by parameter studies. In the calculations, BC 5000 and B1000 silicon oil in reference [3] are chosen and the inclination angle of the substrate is  $45^\circ$ .

This flow has been shown to produce a rich variety of eddies and regions of recirculating flow whose characteristics depend strongly on the topography. Quantitative measures of

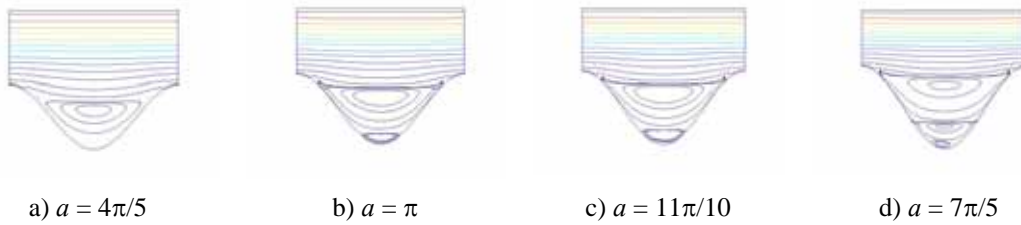


Fig.3 Streamline patterns at  $h = 2\pi$  and  $\lambda=2\pi$  with increasing waviness  $a$

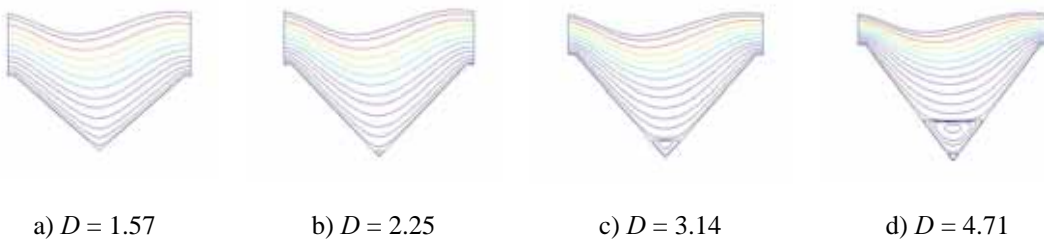


Fig.4 Streamline patterns at  $h = \pi/2$  and  $\lambda=2\pi$  with increasing waviness  $D$

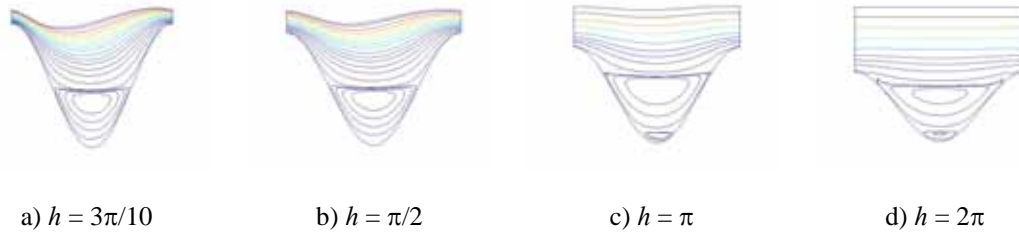


Fig.5 Streamline patterns at waviness  $a = \pi$  and  $\lambda=2\pi$  with increasing film thickness

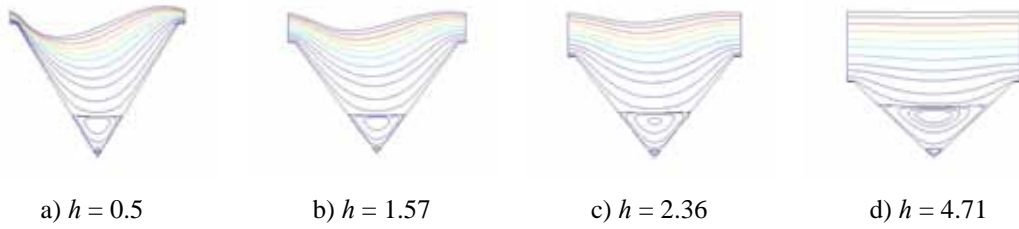
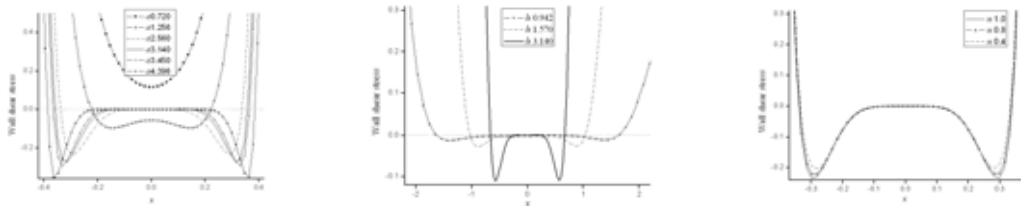


Fig.6 Streamline patterns at  $\lambda=2\pi$  and  $D = 3.14$ ,  $w=5.8$  with increasing film thickness

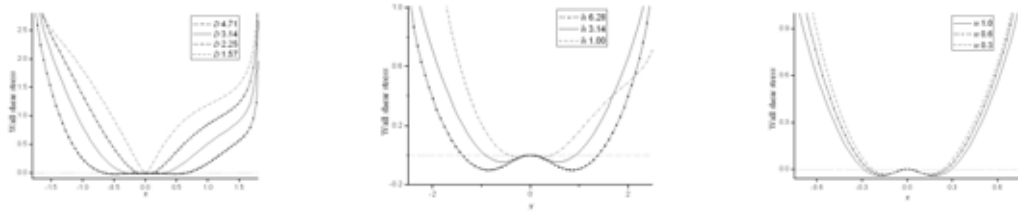
the vortex size as a function of the waviness and film thickness are shown in Fig.3 to Fig.6. The flow separation is also indicated by negative values of the wall shear stress, and the shear stresses variation with waviness  $a$ , film thickness  $h$  and shear-thinning rate  $n$  along the substrate with wavy or triangular profile are shown in Fig.7 and Fig. 8. It also can be seen that the vortices do not apparently change with  $n$ .

It is shown from the above results that for both Newtonian and non-Newtonian flow, the vortices are generated when the waviness of the bottom profile or the film thickness is



a)  $\lambda = 2\pi$ ,  $h=2\pi$  and increasing waviness  $a$     b)  $\lambda = 2\pi$ ,  $a=3.14$  and increasing film thickness  $h$     c)  $\lambda = 2\pi$ ,  $a = 4\pi/5$ ,  $h = 2\pi$  and the decreasing shear-thinning rate  $n$

Fig.7 Bottom shear stress along wavy-plate



a)  $\lambda = 2\pi$ ,  $h=1.57$ ,  $w=5.80$  and increasing waviness  $D$       b)  $\lambda = 2\pi$ ,  $D = 3.14$ ,  $w = 5.80$  and the increasing  $h$       c)  $h = 3.14$ ,  $\lambda = 2\pi$ ,  $D=3.14$ ,  $w=5.80$  and the decreasing shear-thinning rate  $n$

Fig.8 Bottom shear stress along triangular substrate

beyond a critical number. By increasing the waviness or film thickness, a second or third vortex can be generated. The critical waviness and film thickness depend on the wavelength, the amplitude and the inclination angle of the bottom profile

### Acknowledgment

We do appreciate the financial support of National Nature Science Foundation of China.

### References

- [1] P. H. Gaskell, P. K. Jimack, M. Sellier, H. M. Thompson and M. C. T. Wilson. Gravity-driven flow of continuous thin liquid films on non-porous substrates with topography, *Journal of Fluid Mechanics*, 2004, 509, 253-280
- [2] A. Wierschem, N. Aksel. Instability of Liquid Film Flowing down An Inclined Wavy Plane. *Physica D*. 2003, (186): 221~237
- [3] A. Wierschem, M. Scholle, and N. Aksel. Vortices in Film Flow over Strongly Undulated Bottom Profiles at Low Reynolds Numbers. *Physics of Fluids*. 2003, (15): 426~435
- [4] M. Scholle, A. Wierschem, and N. Aksel. Creeping Films with Vortices over Strongly Undulated Bottoms. *Acta Mechanica*. 2004, (168): 167~193