Tracking Vortex Birth (and Death) in Coating Flows

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Introduction

The flows in slot, slide, curtain, roll, blade and some other coating methods are prey to microscopic vortices, gyres, or recirculations that are intense and typically extend across the entire coating bead, as documented by Coyle (1989), Schweizer (1988), Sartor (1990) and others. They tend to centrifuge denser particles, to desorb dissolved gas, to collect and discharge bubbles, to hold formulations long enough for unwanted flocculation or polymerization, and to become nodular along their length and thereby detract from cross-wise coating uniformity.

In computer-aided analysis and design, the most effective approach to these is not to check for them *a posteriori* but to delineate the ranges of design parameters and operating variables in which they are present in a flow. This means tracking the "birth" (and "death") of vortices of two kinds: *free* in the flow, and *attached* to die, blade, or liquid free surface. The means of doing this is to solve the Navier-Stokes or related equation system that governs the flow, after augmenting the system with one or more equations that describe the birth and with an equal number of parameters (dimensionless flow rate, clearance, substrate speed, etc.) as new unknowns.

Theory and computation

Steady coating flows are open flows: they have inflow of liquid supplied and outflow of liquid coated on and carried away by substrate. They are predominantly two-dimensional, because coating uniformity demands that. They may not or they may contain vortices, i.e. closed streamlines nested about a stagnation point. As parameters are changed, a flow lacking a vortex may acquire one. This can happen in two ways: higher-order stagnation point at wall (Figure 1) and appearance of cusp point and evolution from it (Figure 2).

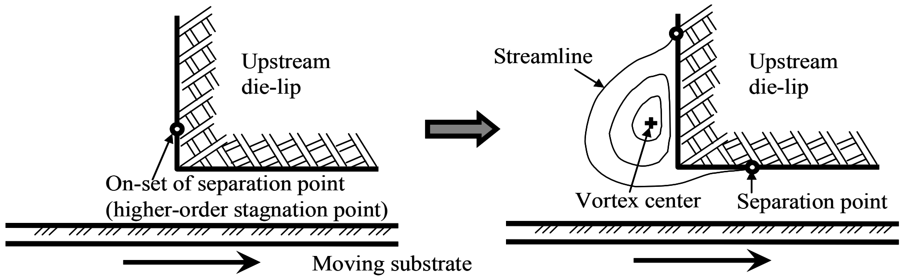

Figure 1. Higher-order stagnation point at wall, i.e. vanishing shear stress: example from slot coating inside slot.

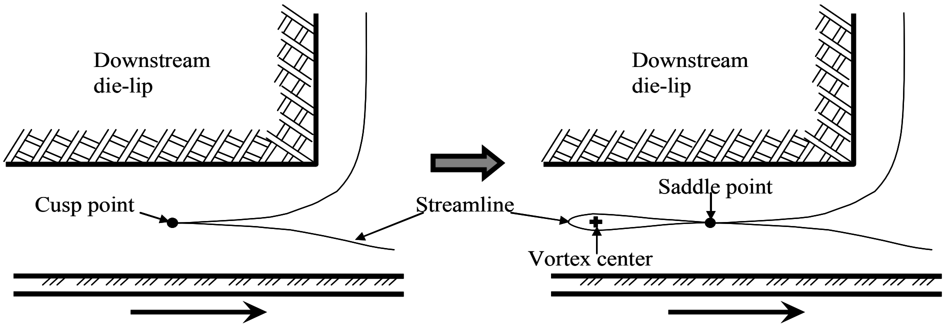

Figure 2. Appearance of cusp and evolution from it: example from slot coating in upstream gap.

Once a vortex has appeared, it can give birth to more vortices by the same process that the primordial vortex of a closed flow like that in a lid-driven cavity does. This case was examined by Gürcan et al.(2003) and similar flow, such as half filled annulus between rotating coaxial cylinders, was examined by Gaskell et al.(2005). Brøns et al.(1999) studied the possible flow structure in a two-dimensional incompressible flow away from boundaries using normal form transformation. Also Bakker (1991) had earlier complete qualitative analysis on bifurcations in flow patterns of steady two-dimensional incompressible viscous flows along the surface of a plane or along a slightly curved wall based on theory of differential equations.

Separation points, cusps, centers of vortices, and saddles between them are all stagnation points. The criteria for birth of vortices, i.e. bifurcation of a stagnation point to new centers, are the determinant of velocity gradient matrix, $\det\mathbf{D}$, must vanish for flow away from boundaries (Brøns et al., 1999) and the shear stress, $\boldsymbol{\tau}_w$, vanishes at wall (Bakker, 1991).

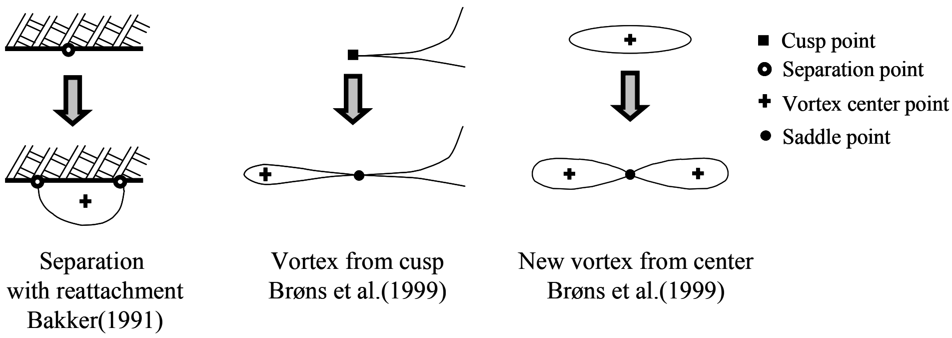

Figure 3. Three types of bifurcation of a stagnation point.

There are two ways to find these special stagnation points in a flow. One is to identify and capture them in solutions of the governing equations as parameters are varied, i.e. by post-processing of solutions, which is straightforward but tedious and costly. The other way is to find exactly parameter combinations at which the stagnation point occurs – a track through parameter space. The procedure is the same as in fold-tracking and other path-following algorithms that make use of continuation in parameters, as by Christodoulou (1990) and Musson (2001). This is the way adopted here: G/FEM Navier-Stokes system are augmented by the stagnation points and their birth criteria: vanishing the determinant of velocity gradient matrix or vanishing shear stress at the wall.

$$[\mathbf{R_u}][\delta\mathbf{u}] = -[\mathbf{R}] \quad \Rightarrow \quad \begin{bmatrix} \mathbf{R_u} & \mathbf{R_p} \\ \mathbf{A_u} & \mathbf{A_p} \end{bmatrix} \begin{bmatrix} \delta\mathbf{u} \\ \delta\mathbf{p} \end{bmatrix} = -\begin{bmatrix} \mathbf{R} \\ \mathbf{A} \end{bmatrix}$$

Newton method on Navier-Stokes G/FEM system — Augmented Navier-Stokes G/FEM system

Figure 4. Augmented Navier-Stokes system for tracking method. 'R' is residual for Navier-Stokes system, 'u' is solution vector, 'A' is residual for augmented system by stagnation point (velocity is zero and birth criterion, '$\det \mathbf{D} = 0$' or '$\boldsymbol{\tau}_w = 0$'), and 'p' is corresponding parameters.

Using tracking method, construction of the "vortex-free" window for a coating system is possible. For example, slot coating has parameters such as dimensionless flow rate, the ratio of gap height to width of feed slot or the ratio of wet-film thickness to gap height. Once these are chosen, the birth of stagnation point is tracked through selected parameters. Typically the "vortex-free" window is smaller than the feasibility window where steady, two-dimensional coating flow is possible.

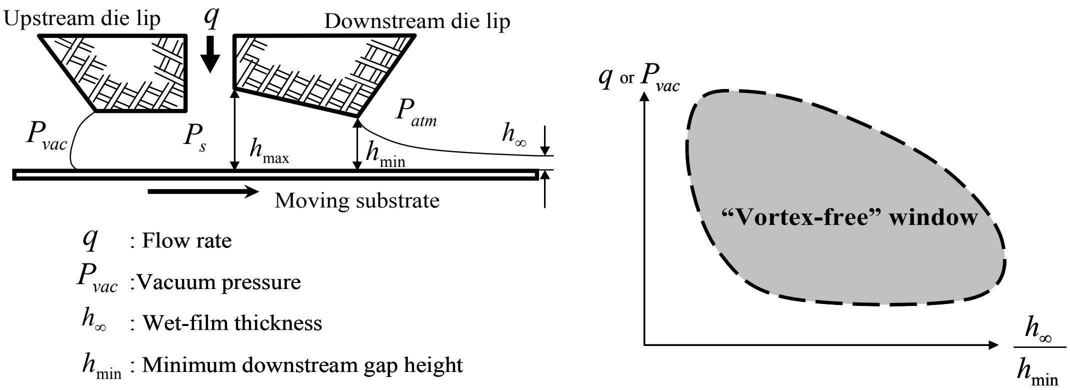

Figure 5. Possible "vortex-free" window for slot coating with two parameters: the ratio of wet-film thickness to minimum downstream gap height and flow rate or vacuum pressure.

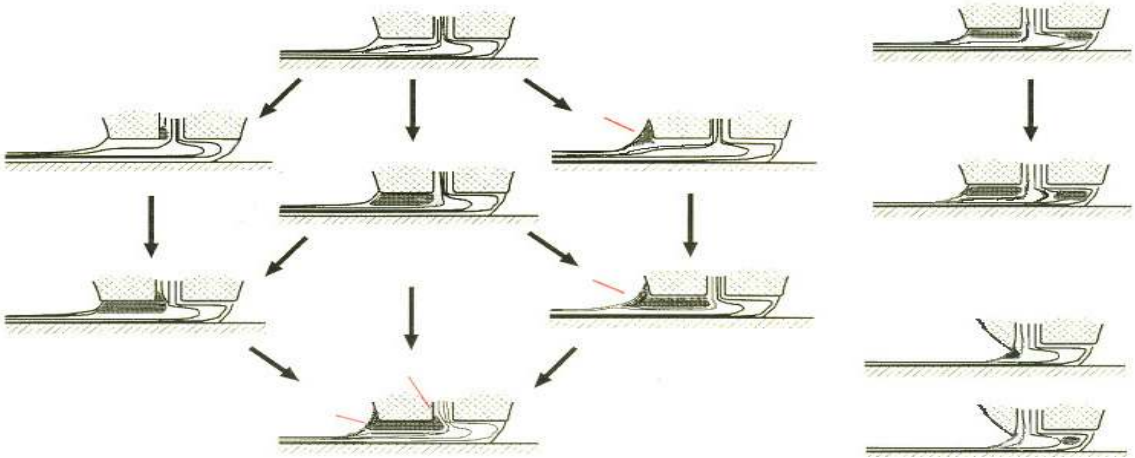

Figure 6. Vortex map of slot coating. Excerpted and modified from Sartor(1990)

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