Coating Flow and De-wetting on Porous Materials

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An economically important class of materials called nonwovens finds application in personal care and medical products, filters and related devices. Liquids are applied either directly to the porous nonwoven material or are deposited on a perforated, but otherwise impermeable, top sheet. We have developed a mathematical and numerical model to predict the subsequent flow history. The mathematical model uses either two-or-three dimensional lubrication theory for the unsteady coating flow on the outer surface, infiltration into the porous pad uses the Washburn equation and spreading within the pad uses Darcy's Law. The model can be used to optimize the design of these products and the effects of changes in surface tension, wettability and permeability can be assessed. Results are particularly relevant to the design of diapers, adult incontinent products, feminine hygiene products, bandages and similar articles.

This work is oriented towards development of a complete process model for pad products. As in other industries that involve the movement of fluids, often practical flow problems involve many different control parameers and it can be very difficult to understand how a final desirable, or undesirable, aspect of product performance is related to its root causes. If a sufficiently accurate computer model can be constructed, process and product optimization can be done first on the computer. Moreover, parameter values can be varied at will, often to values that are difficult or impossible to obtain in products that are currently available. By running the model using various parameter choices, it should be possible to better understand causes and effects, and thus to identify performance improvements.

Pad products typically consist of a porous top sheet, a non-woven absorbing pad, and an impermeable bottom or backing sheet. The so-called "insult" liquid is deposited on the top sheet. The fluid region can be assumed to be long and thin and thus can be treated as a coating flow. The problem may be simplified considerably by invoking the *lubrication approximation*.

Some results of lubrication flow analysis are given first, for an assumed two-dimensional geometry, for flow on the upper or inner surface of the top-sheet. The flux, or total flow within a liquid layer of thickness h, is given by

$$q = \int_0^h u \; dy = -\frac{h^3}{3\mu} \; p_s + \frac{\rho g h^3}{3\mu} \; \sin\theta \tag{1}$$

where u is the flow speed, p is pressure, s is arc length measured along the upper side of the sheet or *substrate* and θ is the downward inclination angle of a particular substrate element from the horizontal. To a first approximation, flow over a curved substrate may be treated simply by "unwrapping" the substrate onto an equivalent straight surface. The magnitude of each of the gravitational force components varies from one element to another. The pressure components remain unaffected by the unwrapping, provided that the fluid layer thickness is much less than the radius of curvature of the substrate.

The pressure includes various contributions, *i. e.*

$$p = -\sigma h_{ss} - \Pi + \rho g h \cos\theta \; . \tag{2}$$

These terms on the right are (i) the contribution from surface tension where h_{ss} approximates the liquid free-surface curvature, (ii) the so-called "disjoining pressure" and (iii) the gravity component perpendicular to the substrate. Subscripts signify differentiation. The disjoining pressure is

$$\Pi = \frac{\sigma\theta_e^2 (n-1)(m-1)}{2h^*(n-m)} \; \left[\left(\frac{h^*}{h}\right)^n - \left(\frac{h^*}{h}\right)^m \right] \tag{3}$$

where θ_e is the local equilibrium contact angle for the liquid on the top-sheet material and h^* is a small constant reference height that represents a fictitious "slip" thickness on "dry" regions of the substrate. It is known to be necessary to include a provision for slip in any dynamic simulation that involves moving contact lines [1]. n and m are positive integers with $n > m$. The results given here use n = 3 and m = 2. The Π term allows incorporation of substrate energetics information.

Overall mass conservation is

$$h_t = -q_s - E \tag{4}$$

where E, having the dimensions of velocity, incorporates liquid removal. The major liquid removal mechanism is absorption to the pad interior but E may also contain evaporation. If E is locally negative, it can include a deposition function representing insult additions as a function of location and time.

Using such dimensionless variables, the single evolution equation to be solved is

$$h_t = (h^3 h_{sss})_s - \frac{C}{h^*}\left(h^3\left[\left(\frac{h^*}{h}\right)^n - \left(\frac{h^*}{h}\right)^m\right]_s\right)_s + \left[h^3(h\cos\theta)_s\right]_s - (h^3\sin\theta)_s - E(s,t) \quad (5)$$

In the dimensionless equation h, s and h^* are measured in units of the so-called capillary length L_c and time is given in units of T^* where

$$L_c = \sqrt{\frac{\sigma}{\rho g}}\,, \qquad T^* = \frac{3\mu L_c}{\sigma}\,. \quad (6)$$

Flow within the pad may be modeled using *Darcy's law*, the standard formalism for flow in porous media. For simplicity the medium can be assumed to be homogeneous and isotropic with constant permeability k. According to Darcy's law, the flow speed is proportional to the pressure gradient; specifically

$$\mathbf{u} = -\frac{k}{\mu}\,\nabla p \quad (7)$$

while mass conservation is

$$\nabla\cdot\mathbf{u} = 0\,. \quad (8)$$

Note that the mass conservation equation would be modified if the model were to allow the pad to swell when liquid is absorbed. Combining these equations, we see that the pressure field within the pad satisfies

$$\nabla^2 p = 0\,. \quad (9)$$

The boundary conditions are that the pressure is essentially atmospheric under the wetted portion of the top-sheet and is less than atmospheric by an amount equal to the capillary pressure at the moving interface within the pad. The capillary pressure depends on the geometry of the closely-spaced fibers and the wetting angle that the invading liquid makes with these fibers. Thus p acts as a velocity potential, allowing the interface to be advanced using (7). As liquid is absorbed, the remaining depth on the top sheet is reduced. No further flow progression is allowed where the interface reaches the backing sheet.

Some results of two-dimensional simulation are shown in Figs (1)-(4). The first three figures are snapshots from shortly after an insult is deposited until it has been absorbed completely in the pad. Perfect wetting and the gravitationally-driven flow allows the insult to spread laterally, both features are seen to promote good absorption. When the same simulation is repeated with less good wetting ($\theta_e = 60^o$), the liquid has contacted the backing sheet and is not absorbed completely after a relatively long time.

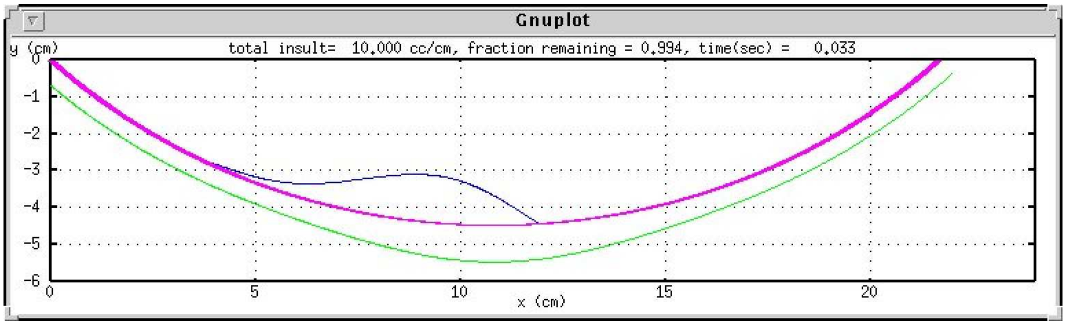

Figure 1: Early-time picture of a flowing insult. Perfect wetting $\theta_e = 0$. Note the residual tail that is left behind. This increase of the size of the "footprint," due to the flow, aids in absorption.

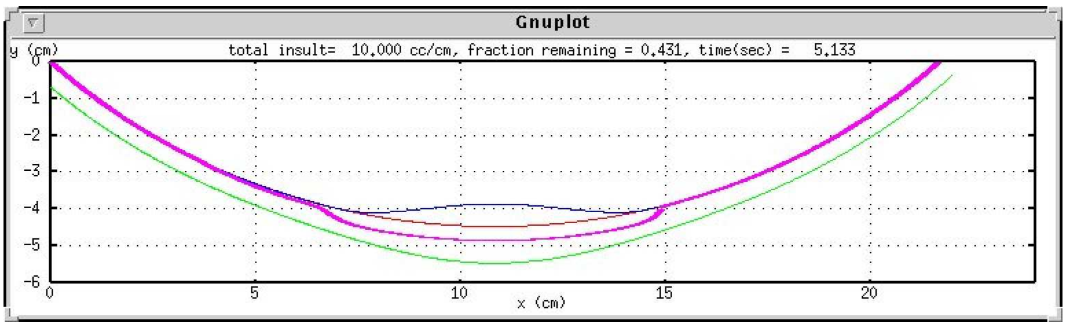

Figure 2: The same insult 5 sec after deposition. It has flowed to the low point and about half of it has been absorbed.

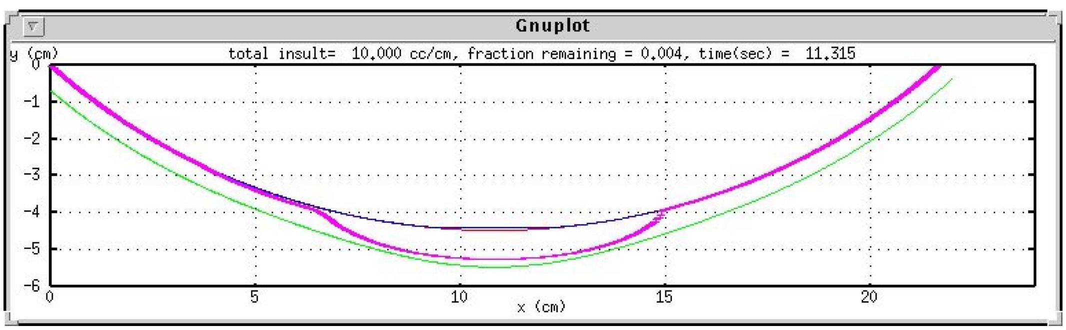

Figure 3: The top sheet is dry at 11 sec after deposition. Notice that the insult has been completely absorbed without ever reaching the impermeable back sheet(the green line).

The two-dimensional analysis given above has been extended so as to model three-dimensional effects. Thus actual deposited volumes can be specified and additional flow effects, such as sideways spreading, can be modeled.

The equations given above remain valid with certain modifications. The mass conservation

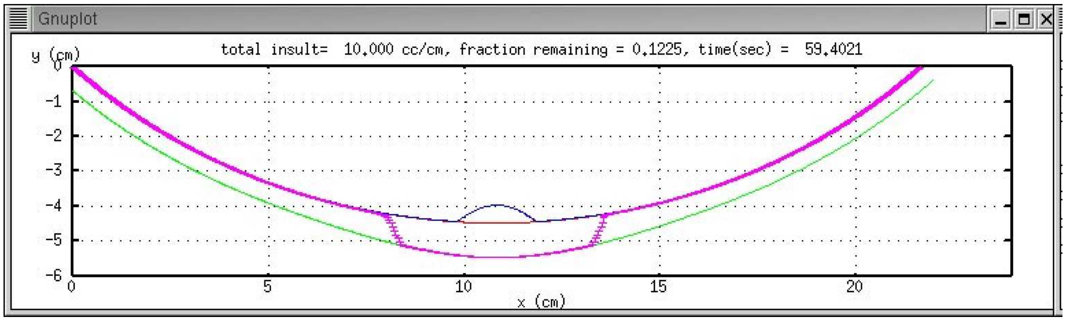

Figure 4: After 59 sec the insult has not been completely absorbed for $\theta_e = 60^o$. The large contact angle causes the insult to have a small footprint. Although the volume is the same as for the perfect wetting case, here the absorbed insult reaches the back sheet. Further absorption can now occur only very slowly since the liquid must spread sideways at the distant boundaries.

equation (4) generalizes to

$$h_t = -\nabla \cdot \mathbf{Q} - E \tag{10}$$

where the ∇ operator is $(\partial/\partial x, \partial/\partial y)$ in the curvilinear top-sheet coordinates (x, y) and **Q** is a flux *vector*. The top sheet is assumed to be a ruled surface with curvature only in the (x, z) plane. The direction of gravity is parallel to the (x, z) plane. Thus y is a Cartesian coordinate "out of the paper"while x is arc-length along the bent top sheet. x is essentially similar to the arc length s used above. z is distance measured perpendicular to the top-sheet. Thus, for example, the liquid free surface is given by $z = h(x, y, t)$. There is a pad of specified thickness below the topsheet whose thickness equation $z = T_{pad}(x, y)$. Only a pad of constant thickness is considered in the 3D simulation shown here. The saturation interface within the pad, which gives the shape of the soaked pad region, is denoted by the function $z = -Z(x, y, t)$.

The evolution equation for the free liquid surface above the pad becomes

$$h_t = -\nabla \cdot \frac{\sigma h^3}{3\mu} \left[\nabla \nabla^2 h - \frac{\rho g}{\sigma} \nabla \left(h \cos\theta \right) + \frac{1}{\sigma} \nabla \Pi \right] - \frac{\rho g}{3\mu} \frac{\partial}{\partial x} \left(h^3 \sin\theta \right) - E \tag{11}$$

where $\theta = \theta(x)$ is the local topsheet inclination relative to the direction of gravity. The topsheet is currently assumed to be bent into a circular arc. The disjoining function Π and the specification of equilibrium contact angle θ_e are as given in equation (3).

A relatively simple pad flow model has been used here. It consists of two parts. There is normal infiltration; specifically equation (7) is replaced by the simpler Washburn-type relation

$$\frac{dZ}{dt} \propto \frac{1}{h^* + Z} \; . \tag{12}$$

Then sideways spreading within the pad is modeled using a diffusion equation for $Z(x, y, t)$,

$$Z_t = \kappa \, \nabla^2 Z \tag{13}$$

where the constant κ can be best-fitted using experimental measurements.

Simulation results for a time at which about half the original insult has been absorbed are given in Figs (5) - (7). A wire-cage picture of the remaining insult surface is shown in Fig (5); alternatively a contour plot can be constructed as in Fig. (6). The absorption profile within the pad is shown in Fig. (7).

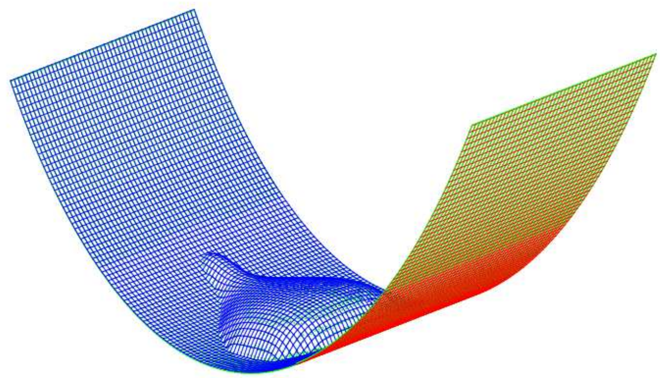

Figure 5: A frame from a 3-dimensional simulation showing a flowing mound of liquid on the top sheet.

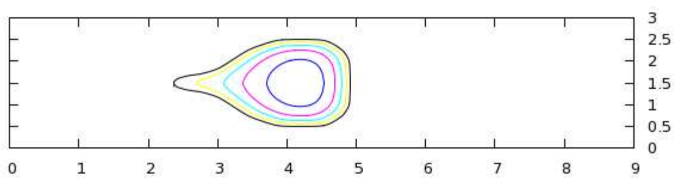

Figure 6: A contour plot of the top sheet liquid distribution using the same data as the previous picture. The rectangular pad is of dimensions 9 cm by 3 cm.

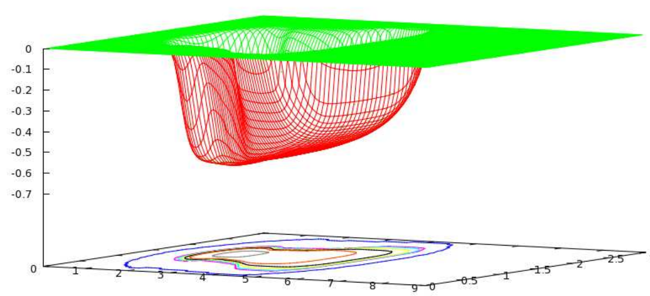

Figure 7: The adsorption distribution in the pad. A small amount of dispersive spreading within the pad is incorporated in the simulation. For ease of display, the pad is "unwrapped." The space scales are in cm while the vertical scale Z is dimensionless. The top sheet corresponds to $Z = 0$ and the impermeable backing is $Z = -1$.

A number of other features have been incorporated into the model. The actual speed of liquid flow into the pad, at each point, is controlled by the size, shape, and spacing of the holes in the top sheet. The computer model allows these inputs to be specified. The flow of the insult along the top sheet is strongly influenced by the local equilibrium contact angle. Typically the top sheet is impregnated with a surfactant or wetting agent. It is known that this wetting agent loses some effectiveness as a function of contact time with liquid. The model can accept variation of contact angle with position and time. The top-sheet holes typically are truncated elliptical cones (as seen in Fig. 8) and an approximate treatment of this feature is also included.

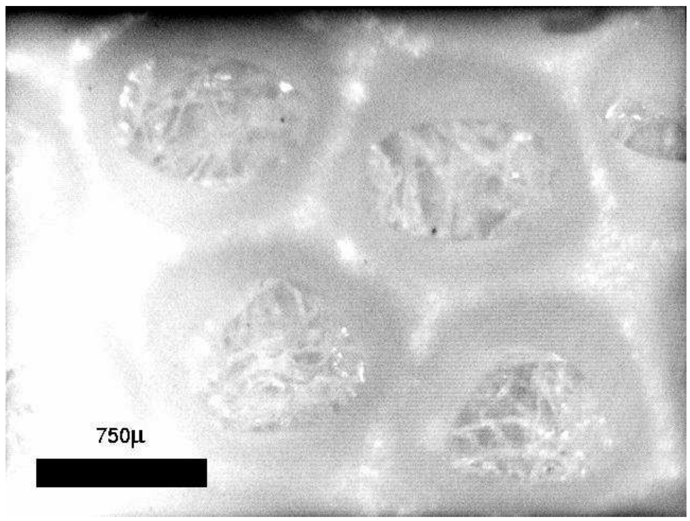

Figure 8: Micrograph of topsheet showing some pad fibers through the cones. The length of the bar in the figure is 750 μm.

References

[1] Huh, C. & Scriven, L. E., Hydrodynamic model of steady movement of a solid/liquid/fluid contact line, *J. Colloid & Interf. Sci.* 35, 85 - 101, 1971.