Spin Coating of a Thin Conducting Liquid Film in the Presence of a Magnetic Field: Dynamics and Stability

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1. INTRODUCTION

The flow of a thin film of viscous fluid over a smooth rotating disk has attracted the attention of several investigators in science and engineering due to its enormous applications in many industrial processes that range from the intensification of heat and mass transfer processes in chemical reactors to powder production in metallurgy. The production of thin films on substrates placed in the grooves of a rotating disk is refereed to as 'Spin Coating' in literature and this technique is employed in coating a very thin and uniform film of photoresist on silicon wafers for integrated circuits or of a layer of very thin magnetic paint on the substrate, magnetic storage disks, fabrication on thin uniform layers of plastic scintillator on supporting aluminized mylar and so on.

In spite of the difficulties in modeling this flow mathematically due to the variation of acceleration along the radius, the flow over a spinning disk has lent itself more naturally to potential technological exploitation due to the possibility of controlling the local accelerations. The final thickness of the film and the uniformity in the thickness are central issues in these applications and they are observed to be influenced by several factors such as the viscosity of the liquid film, different spin-up protocols, heat and mass transfer processes and so on.

Since the pioneering study by Emslie *et al.* (1958) on the hydrodynamic analysis of the flow of a Newtonian fluid on a spinning disk, a number of theoretical and experimental studies of the spin-coating process that include modeling of flow over a rotating disk, wave generation in the liquid film moving on the surface of a rotating disk and the stability characteristics of a thin film on a rotating substrate, have been reported (Reisfeld *et al.*, 1991; Kitamura, 2001; Usha *et al.*, 2005). The presence study addresses the dynamics and stability characteristics of a viscous conducting film over a spinning disk in the presence of a transverse magnetic field. It is observed that infinitesimal disturbances decay for small wave numbers for different values of Hartmann number considered and that they are transiently stable for larger wave numbers.

2. MATHEMATICAL FORMULATION

Consider a film of viscous conducting fluid on a rotating disk. A system of cylindrical coordinates (r, θ, z) that rotates with the disk at an angular velocity Ω about the z-axis is used, where r measures the radial distance from the center of the disk, θ is the angle from some fixed radial line in the horizontal plane and z measures the distance vertically upward from the solid surface of the disk. A uniform magnetic field B_0 acts parallel to the axis of rotation of the disk (Figure 1). The liquid-gas interface is located at $z = h(r, t)$, where h is the film thickness as a function of r and t. The non-dimensional governing equations and boundary conditions are

$$\epsilon Re\left[u_t + uu_r + wu_z\right] - \epsilon^2 Re^2 \frac{v^2}{r} = -p_r + r + 2\epsilon Rev + \epsilon^2\left[u_{rr} + \frac{u_r}{r} - \frac{u}{r^2}\right] + u_{zz} - M^2 u \quad (1)$$

$$\epsilon Re\left[v_t + uv_r + \frac{uv}{r} + wv_z\right] = \epsilon^2\left[v_{rr} + \frac{v_r}{r} - \frac{v}{r^2}\right] + v_{zz} - 2u - M^2 v \quad (2)$$

$$\epsilon^3 Re\left[w_t + uw_r + ww_z\right] = -p_z + \epsilon^4\left[w_{rr} + \frac{w_r}{r}\right] + \epsilon^2 w_{zz} - \frac{\epsilon Re}{F^2}; \quad u_r + \frac{u}{r} + w_z = 0 \quad (3)$$

$$u = v = w = 0 \text{ on } z = 0; (-h_t - uh_r + w)(1 + \epsilon^2 h_r^2)^{-1/2} = \frac{2E}{3} \text{ on } z = h(r, t) \quad (4)$$

$$-p + 2\epsilon^2 (1 + \epsilon^2 h_r^2)^{-1}\left[\epsilon^2 u_r h_r^2 + w_z - u_z h_r - \epsilon^2 w_r h_r\right] = \frac{\epsilon We}{r(1 + \epsilon^2 h_r^2)^{3/2}}\left[h_r + \epsilon^2 h_r^3 + rh_{rr}\right] \text{ on } z = h(r, t) \quad (5)$$

$$2\epsilon^2 h_r (w_z - u_r) + (1 - \epsilon^2 h_r^2)(u_z + \epsilon^2 w_r) = 0; \quad v_z - \epsilon^2 h_r\left(v_r - \frac{v}{r}\right) = 0 \text{ on } z = h(r, t) \quad (6)$$

where $\epsilon = h_0/L$ is the aspect ratio, $Re = U_0 h_0/\nu$ is the Reynolds number, $F = \sqrt{U_0^2/gh_0}$ is the Froude number, $We = \sigma/\rho\Omega^2 L h_0^2$ is the Weber number and M is the Hartmann number given by $M^2 = \sigma B_0^2 h_0^2/\mu$. The dependent variables u, v, w and p are expanded in powers of ϵ as $(u, v, w, p) = \sum_{n=0}^{N} \epsilon^n \left[u^{(n)}, v^{(n)}, w^{(n)}, p^{(n)}\right]$ and substituted in equations (1) - (6) and the resulting zeroth and first order equations are solved and the evolution equation is obtained using (4) as

$$h_t + \frac{2}{3}E + \frac{2h}{M^2} - \frac{2\sinh Mh}{M^3\cosh Mh} + \frac{rh_r\sinh^2 Mh}{M^2\cosh^2 Mh} + \epsilon Re\left\{\frac{14\sinh Mh}{3M^7\cosh^3 Mh} + \frac{61\sinh 2Mh}{6M^7\cosh^2 Mh}\right.$$
$$- \frac{13h}{3M^6\cosh^2 Mh} - \frac{2h}{M^6\cosh^4 Mh} - \frac{10h}{M^6} - \frac{4h^2\sinh Mh}{M^5\cosh^3 Mh} + \frac{4E(\cosh Mh - 1)}{3M^4\cosh^2 Mh}$$
$$\left. - \frac{2Eh\sinh Mh}{3M^3\cosh^3 Mh}\right\} + \epsilon Re(rh_r)\left\{\frac{12h\sinh Mh}{M^5\cosh^3 Mh} - \frac{5}{M^6} - \frac{11}{2M^6\cosh^2 Mh} + \frac{21}{2M^6\cosh^4 Mh}\right.$$
$$+ \frac{4h^2}{M^4\cosh^2 Mh} - \frac{6h^2}{M^4\cosh^4 Mh} + \frac{11h\sinh Mh}{2M^5\cosh^4 Mh} - \frac{2E\sinh Mh}{3M^3\cosh^2 Mh}$$

$$+\frac{E\sinh Mh}{M^3\cosh^3 Mh}+\frac{2Eh}{3M^2\cosh^2 Mh}-\frac{Eh}{M^2\cosh^4 Mh}\Bigg\}$$
$$+\epsilon Re(r^2h_r^2)\left\{-\frac{11\sinh Mh}{2M^5\cosh^5 Mh}-\frac{2h}{M^4\cosh^2 Mh}+\frac{h}{M^4\cosh^4 Mh}+\frac{5h}{2M^4\cosh^6 Mh}\right.$$
$$\left.+\frac{4\sinh Mh}{M^5\cosh^3 Mh}\right\}+\epsilon Re(r^2h_{rr})\left\{-\frac{3\sinh^2 Mh}{2M^6\cosh^4 Mh}+\frac{h\sinh Mh}{M^5\cosh^3 Mh}+\frac{h\sinh Mh}{2M^5\cosh^5 Mh}\right\}=0 \quad (7)$$

The evolution equation is solved numerically using Crank-Nicolson finite-difference scheme and the results for the film thickness and the influence of magnetic field on the rate of thinning will be discussed during th talk. In what follows, the stability characteristics of (7) describing the shape of the film thickness as a function of space and time is examined using linear theory.

3. DESCRIPTION OF TIME-DEPENDENT BASIC STATE

As the film is draining due to centrifugation, the basic state is time dependent and it is assumed to be flat. The film thickness is independent of the radial position and the governing base state behaviour is obtained as

$$\bar{h}_t + \frac{2}{3}E+\frac{2\bar{h}}{M^2}-\frac{2\sinh M\bar{h}}{M^3\cosh M\bar{h}}+\epsilon Re\left\{\frac{14\sinh M\bar{h}}{3M^7\cosh^3 M\bar{h}}+\frac{61\sinh 2M\bar{h}}{6M^7\cosh^2 M\bar{h}}-\frac{13\bar{h}}{3M^6\cosh^2 M\bar{h}}\right.$$
$$\left.-\frac{2\bar{h}}{M^6\cosh^4 M\bar{h}}-\frac{10\bar{h}}{M^6}-\frac{4\bar{h}^2\sinh M\bar{h}}{M^5\cosh^3 M\bar{h}}+\frac{4E(\cosh M\bar{h}-1)}{3M^4\cosh^2 M\bar{h}}-\frac{2E\bar{h}\sinh M\bar{h}}{3M^3\cosh^3 M\bar{h}}\right\}=0 \quad (8)$$

with $\bar{h}(0)=1$. Figure 2 shows the basic state film thickness for principal values of the evaporation parameter E and the Hartmann number M. It is observed that film thickness decreases monotonically as time goes to ∞. The effect of Hartmann number M is to enhance the film thickness irrespective of the value of evaporation parameter E. Evaporation and drainage are the causes for thinning of the fluid layer when $E>0$. As Hartmann number increases, the film takes a longer time to thin down to a prescribed height.

4. LINEAR STABILITY ANALYSIS AND DISCUSSION

The linearised disturbance equation is obtained from equation (7) by perturbing the basic state by a small amount η ($h=\bar{h}(t)+\delta\ \eta(r,t)$) and substituting in (7) and linearising in disturbance amplitude 'δ' as

$$\eta_t+a_1(t)r^2\eta_{rr}+a_2(t)r\eta_r+a_3(t)\eta=0 \quad (9)$$

$$a_1(t) = \epsilon Re\left\{-\frac{3\sinh^2 M\bar{h}}{2M^6\cosh^4 M\bar{h}}+\frac{\bar{h}\sinh M\bar{h}}{M^5\cosh^3 M\bar{h}}+\frac{\bar{h}\sinh M\bar{h}}{2M^5\cosh^5 M\bar{h}}\right\} \quad (10)$$

$$a_2(t) = \frac{\sinh^2 M\bar{h}}{M^2\cosh^2 M\bar{h}}+\epsilon Re\left\{\frac{12\bar{h}\sinh M\bar{h}}{M^5\cosh^3 M\bar{h}}-\frac{5}{M^6}-\frac{11}{2M^6\cosh^2 M\bar{h}}\right.$$
$$+\frac{21}{2M^6\cosh^4 M\bar{h}}+\frac{4\bar{h}^2}{M^4\cosh^2 M\bar{h}}-\frac{6\bar{h}^2}{M^4\cosh^4 M\bar{h}}+\frac{11\bar{h}\sinh M\bar{h}}{2M^5\cosh^4 M\bar{h}}-\frac{2E\sinh M\bar{h}}{3M^3\cosh^2 M\bar{h}}$$

$$+\frac{E\sinh M\bar{h}}{M^3\cosh^3 M\bar{h}}+\frac{2E\bar{h}}{3M^2\cosh^2 M\bar{h}}-\frac{E\bar{h}}{M^2\cosh^4 M\bar{h}}\Bigg\} \tag{11}$$

$$\begin{aligned} a_3(t) \;=\; & \frac{2}{M^2}-\frac{2\sinh M\bar{h}}{M^3\cosh M\bar{h}}(\coth M\bar{h}-\tanh M\bar{h}) \\ & +\epsilon Re\left\{\frac{14\sinh M\bar{h}}{3M^7\cosh^3 M\bar{h}}(\coth M\bar{h}-3\tanh M\bar{h})\right. \\ & -\frac{13}{3M^6\cosh^2 M\bar{h}}(1-2\bar{h}\tanh M\bar{h})+\frac{61\sinh 2M\bar{h}}{6M^7\cosh^2 M\bar{h}}(\coth 2M\bar{h}-2\tanh M\bar{h})-\frac{10}{M^6} \\ & -\frac{2}{M^6\cosh^4 M\bar{h}}(1-4\bar{h}\tanh M\bar{h})-\frac{4\sinh M\bar{h}}{M^5\cosh^3 M\bar{h}}(2\bar{h}+\bar{h}^2\coth M\bar{h}-3\bar{h}^2\tanh M\bar{h}) \\ & \left.-\frac{4E\tanh M\bar{h}}{3M^4\cosh M\bar{h}}+\frac{8E\tanh M\bar{h}}{3M^4\cosh^2 M\bar{h}}-\frac{2E\sinh M\bar{h}}{3M^3\cosh^3 M\bar{h}}(1+\bar{h}\coth M\bar{h}-3\bar{h}\tanh M\bar{h})\right\} \end{aligned} \tag{12}$$

Assuming η in the form $\eta(\xi,t)=H(t)e^{ik_0\xi}$, the equation for normal mode amplitude $H(t)$ is obtained as

$$\frac{\dot{H}}{H}=a_1(t)\frac{k^2}{\epsilon}-[a_2(t)-a_1(t)]\frac{i\;k}{\sqrt{\epsilon}}-a_3(t) \tag{13}$$

where $k=\sqrt{\epsilon}k_0$. The extremum point in time for $|H|$ occurs whenever $a_1(t)k^2/\epsilon-a_3(t)=0$ which gives $k_c^2=\epsilon\; a_3(t)/a_1(t)$ where k_c is cut off wave number, which increases with increase in Hartmann number.

Figure 3 presents the normal mode amplitude for a disturbance which is (a) Stable - ($k^2<k_c^2$), (b) Transiently stable - ($k^2=k_c^2$) and (c) Unstable - ($k^2>k_c^2$) for Hartmann number $M=0$ and 2 and evaporation parameter $E=0$. Since the cut off wave number k_c increases with the increase in Hartmann number, the effect of magnetic field is to enhance the stability of a thin non-uniform conducting film flow on a rotating disk.

Figure 4 shows the disturbance amplitude as a function of time for $Re=6.0$ when $k=4.5;\ M=0$ and $k_c=19.5; M=2$ for the transiently stable behaviour. It is observed that $|H/H_0|$ exhibits two regions - one in which the disturbance is stable and the other in which it is transiently stable. The disturbance amplitude increases initially, reaches a maximum and then decays monotonically. The disturbance amplitude takes a value below its initial value as time increases.

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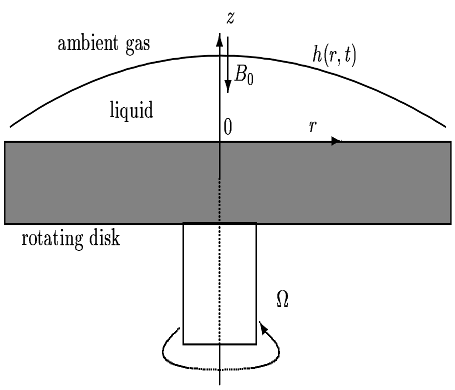

Figure 1. Schematic representation of film flow on a heated rotating disk.

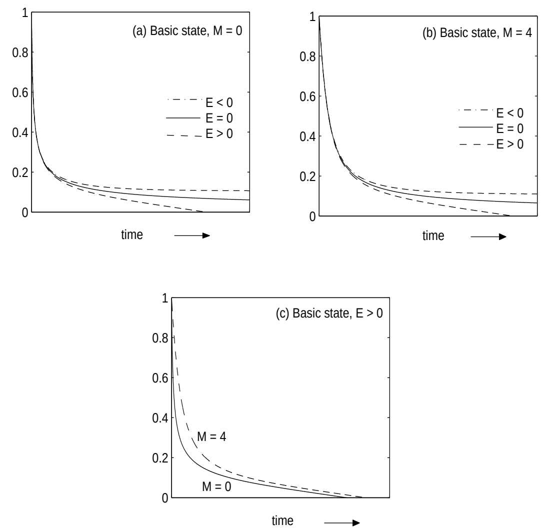

Figure 2. Basic state film thickness for principal values of the evaporation parameter and different values of Hartmann number.

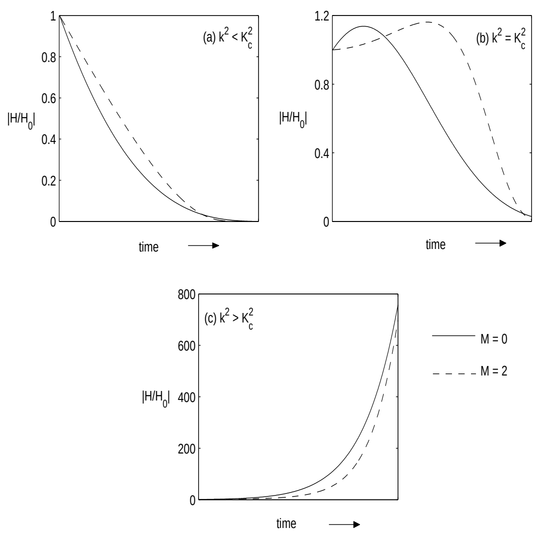

Figure 3. Normal mode amplitude for a disturbance when $Re = 6.2$, $\epsilon = 0.01$, $E = 0$ and $M = 0,\ 2$. (a) Stable (b) Transiently stable (c) Unstable.

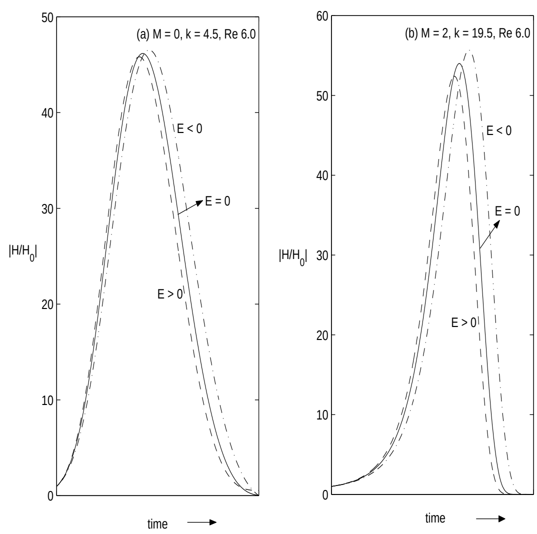

Figure 4. Normalized disturbance amplitude over time when $Re = 6.0$, $\epsilon = 0.01$, $E = 0,\ 0.0012, -0.0012$ and $M = 0,\ 2$.