Film dynamics on a vertically rotating disk partially immersed in a liquid bath

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Abstract

The spreading of a thin film on a single circular disk rotating about horizontal axis and partially immersed in the reservoir with the liquid represents a prototype used in many industrial applications, such as PET polycondensation reactors, oil disk skimmers for water purification, blood oxygenator and tissue engineering. For the small capillary numbers and Newtonian liquids we derived a dimension reduced generalized lubrication approximation. For this two-dimensional nonlinear degenerate fourth-order boundary value problem we developed a finite element scheme that captures the evolution of the film profile on the complete disk. For a range of parameters we found steady state solutions. Correspondingly, we performed an asymptotic analysis near the meniscus region and a careful comparison with cross sections of the numerical solutions along constant radii gave excellent agreement. The more complicated high capillary number and shear-thinning liquid cases studied numerically and analytically for the simplified problem of withdrawal of a plate from a liquid bath. The dimension-reduced lubrication equations are derived for the most commonly used shear-thinning viscosity models. A careful phase plane analysis then shows the existence of two types of solutions. Type I corresponding to a monotone film profile and Type II, corresponding to a spatially oscillating film profile. We develop criteria for the power-law and Ellis model that select the type of solution.