Linear stability and nonlinear dynamics in particle-laden thin films

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We study a thin film of viscous fluid containing a suspension of non-brownian spherical particles. Experiments by Zhou et al. in 2005 [4] found new behaviors in this type of flow not seen in Newtonian films, involving visible phase segregation. At high inclination angles and high solid volume fractions (greater than about 40%), particles were observed to accumulate in a thick ridge at the advancing contact line. The well-known fingering instability at the contact line [2] was also found to occur at longer wavelengths and grow more slowly when this ridge was present. A lubrication model describing the formation of this ridge was originally proposed by Zhou et al. and refined in [1], and a version of this model neglecting surface tension was found to have double-shock solutions in qualitative agreement with the observed ridge.

Essential features of the lubrication model include the assumption that the particle concentration $\phi(x, y, t)$ is independent of the coordinate z normal to the inclined plane, thus permitting the Stokes equations to be depth-averaged as for pure liquid films, and the use of a mixture viscosity $\mu(\phi) = (1-\phi/\phi_m)^{-2}$, which immobilizes the mixture as the the concentration reaches a value ϕ_m representing close packing. Phase segregation is incorporated through a settling velocity v_{rel} which is expressed as a function of ϕ and the film depth h.

This work explores the contact line instability of particle-laden films through numerical solutions of the full fourth-order model equations in two dimensions. These solutions exhibit two unanticipated oscillations in the downstream x and transverse y directions, the latter terminating in numerical instability. This behavior is due to the fact that while the fourth-order

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surface tension terms of the model tend to regularize h, ϕ is effectively unregularized. We therefore add a small diffusion term to the model which damps these oscillations, and discuss possible physical interpretations.

We study the linear stability of the contact line by numerically computing the growth rate of perturbations of the form $\bar{h}(x,y,t) = h(x,t) + \epsilon g(x,t)\cos qy$, $\bar{\phi}(x,t) = \phi(x,t) + \epsilon\psi(x,t)\cos qy$. The equations display a long wave instability much like that seen in pure fluid films [3], and the wavenumbers q of fastest growth are seen to shift to slightly longer waves compared to the pure fluid case. The maximum growth rate also appears to be slightly reduced, though this effect is nearly imperceptible. Thus while these findings are in qualitative agreement with the observations of Zhou et al., it is not clear that their magnitude is sufficient to explain the experimental effects.

References

[1] B. Cook, A.L. Bertozzi, and A.E. Hosoi. A lubrication model for particle-laden thin films. *SIAM J. Appl. Math.* Submitted 2006.

[2] Herbert E. Huppert. Flow and instability of a viscous current down a slope. *Nature*, 300:427–429, Dec 1982. 10.1038/300427a0.

[3] S.M. Troian, E. Herbolzheimer, S.A. Safran, and J.F. Joanny. Fingering instabilities of driven spreading films. *Europhysics Letters*, 10:25, September 1989.

[4] J. Zhou, B. Dupuy, A.L. Bertozzi, and A.E. Hosoi. Theory for shock dynamics in particle-laden thin films. *Phys. Rev. Lett.*, 209(11), 2005.