

Numerical investigation of the linear response of a viscous liquid sheet

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Abstract

Thin viscous liquid sheets, as used e.g. in the curtain coating process, are sensitive to external disturbances, such as pressure fluctuations along their interfaces. The paper to be presented investigates the propagation of such disturbance signals in a uniform thin viscous liquid sheet of infinite extent which is in contact with a passive ambient medium. The disturbances are induced by local external pressure perturbations moving with constant velocity $\vec{V}$ along the sheet. The tool of analysis is the Fourier-Laplace transformation of the linearized perturbation equations, and the inverse Fourier-Laplace transform

$$\eta_{v,s}(\vec{x}, t) = -\frac{1}{2\pi} \int d\vec{k} e^{i\vec{k} \cdot \vec{x}} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\omega e^{\omega t} \frac{k \tilde{\mathcal{P}}(\vec{k}, \omega)}{D_{v,s}(k, \omega)}, \quad c > 0, \quad (1)$$

for the reconstruction of the signal, the amplitude of the interface deflections². The response is determined by the shape of the disturbance, $\mathcal{P}(\vec{x}, t)$, and by the intrinsic response properties of the viscous sheet, codified in its two dispersion functions

$$D_v(k, \omega) = \Gamma^2 k^3 + (k^2 + l^2)^2 \coth(k) - 4k^3 l \coth(l), \quad (2)$$

$$D_s(k, \omega) = \Gamma^2 k^3 + (k^2 + l^2)^2 \tanh(k) - 4k^3 l \tanh(l), \quad (3)$$

with $\vec{k}$ the two-dimensional wave vector, $k = |\vec{k}|$, and $l^2 = k^2 + \omega$, and with the characteristic number $\Gamma = \sqrt{\gamma H / (2\rho\nu^2)}$, which determine the singularities of the Fourier-Laplace integrand. Symmetric (varicose) and antisymmetric (sinuous) disturbances are investigated in the long time limit by numerical signal evaluation¹. Fig. 1 shows for illustration in the rest frame of the perturbation source the varicose and sinuous response of the sheet to a localized pressure perturbation moving with the dimensionless velocity $V = 2\Gamma$ with respect to the sheet.

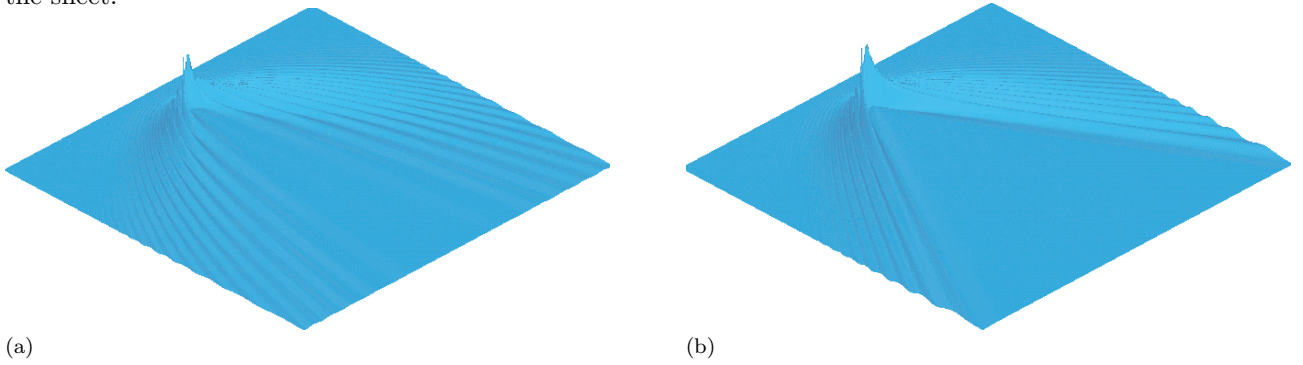


Figure 1: Propagation of disturbances for a moving external pressure profiles $P_s(\vec{x}, t)$, $\Gamma = 100$, $V = 2\Gamma$ (shown in the rest frame of disturbance): Numerical results for a varicose disturbance (left, dispersion function eq. (2)), and eq. (3) for a sinuous disturbance (right, dispersion function eq. (3)).

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References

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