Stokes versus Darcy interfacial flows: the case of suction

Martine BEN AMAR

Laboratoire de Physique Statistique,

Ecole Normale Supérieure , 24, rue Lhomond 75231 Paris France

June 19, 2007

Interfacial instabilities have been the subject of numerous experimental and theoretical works in the past [1]. They mimic pattern formations and in the case of Laplacian growth [2], they are well understood. Recently, experiments involving suction in Hele-Shaw cells have shown possible discrepancies between experimental data and theoretical predictions. The experiment consists in the lifting of the upper plate of the Hele-Shaw cell which induces a decrease of the horizontal size of the viscous blob. At short times, just after the beginning of the suction process, a border instability occurs with a large number of tiny undulations. The instability known as the Saffman-Taylor viscous fingering instability in another geometry is the result of the competition between the capillary resistance and the hydrodynamic flux induced by suction. The smallness of the undulations allows a linear perturbation analysis, rather easy to perform for Darcy flows (Laplacian flows) which can be extended to power-law flows [3]. It turns out that , as very often in hydrodynamics, the instability is easily explained but occurs for a range of possible wavelengths. Some wavelength selection criteria have been given in the past, based on physical arguments without a real check with experimental data. For the experiment of Lindner et al. [4], the discrepancy is enough significant to re examine the wave-length selection of these patterns.

Another (independent) experiments of suction has been made in lipidic membranes with rafts (experiment of M. Angelova group). Here, the suction is induced by HDL complex molecules which attack the cholesterol of the raft domains , leading to a collapse of the raft phase. Depending on the HDL concentration, the raft or liquid-ordered domain which is initially circular collapses with a circular shape or with undulations. The number of undulations is less than ten in this experiment but the origin seems to be the same: that is a suction instability. Nevertheless, in the context of lipid membranes, the simplest model for hydrodynamics is the two-dimensional Stokes flow since the lipid displacements are confined to the surface of a huge sphere, compared to their size. It turns out that the suction process for a two-dimensional Stokes flow has received much less attention, probably because of the difficulty to realize a true 2D-flow experimentally but also because of the difficulties [5, 6] of the calculations for a physical answer that everybody expects to be similar to the Laplacian flow. Very surprisingly, the Stokes instability of suction differs significantly from the Darcy instability [7]. It occurs only for convergent radial flow independently of the sign of the viscosity contrast between the two phases involved. The dispersion relation is also unusual and we cannot apply the qualitative criteria of Laplacian flows to deduce a possible wavelength. It is why, for these two instabilities of suction, we perform a selection analysis based on a variational procedure taking into account the balance of capillarity and the viscous stresses to predict the correct number of oscillations. For both flows, Darcy or Stokes, we are able to predict quantitatively the pattern wavelength. The range of application of this result is broad and experimental verification rather easy. It may have some importance, especially as soon as biological processes are involved. As a consequence it allows to determine some physical quantities such as line tension or viscosities which are not easy to measure in biological membranes.

References

[1] Homsy G.M.,Annual Review fluid mech. **19**271(1987),Couder Y.,Chaos,Order and patterns (Plenum Press, New York) (1991).

[2] Tanveer S, Journ. Fluid. Mech. **409**, 273 (2000).

[3] M. Ben Amar, D. Bonn, Physica D **209**, 1 (2005)

[4] A. Lindner, D. Derks, M. J. Shelley, Phys. Fluids **17**, 072107 (2005)

[5] Crowdy DG, Tanveer S, Journ.of Nonl. Sciences 8 (3): 261-279 (1998)

[6] L. J. Cummings, S. D. Howison and J. King, Eur. Journ. Appl. Math. **10**, 635 (1999)

[7] Paterson L. ,Journ. Fluid. Mech **113**, 513 (1981)