## TIME INTEGRATION AND PATH-FOLLOWING APPLIED TO LUBRICATION EQUATIONS FOR 3D PROBLEMS

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ABSTRACT. Pattern formation in thin liquid films represents a highly nonlinear phenomenon far from equilibrium. Its study requires a numerical treatment of the fully nonlinear system allowing for time integration of the dynamics and path-following to directly track equilibria. We present a code unifying both tasks for lubrication-type equations in analogy to a similar approach for the Navier-Stokes equations [1]. We show that time-stepping based on an *exponentiation propagation* scheme is much better adapted to the lubrication equation than the classically used semi-implicit scheme, especially for the automatic adaptation of the timestep. The developed common numerical framework is applied to the three-dimensional phenomena of

- (1) evolving patterns in dewetting, modelling initial 'rupture' via surface instability or nucleation followed by a slow coarsening;
- (2) stable sliding drops on an inclined homogeneous substrate and the transition to sliding drops that emit secondary droplets;
- (3) stick-slip motion of drops on a heteregeneous substrate and the rich depinning scenarios of ridges and drops.

## References

 Tuckerman, L. and Barkley, D. Bifurcation analysis for timesteppers Numerical Methods for Bifurcation Problems and Large-Scale Dynamical Systems, Springer, New York, Doedel, E. (Ed) 2000.

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