# Surface instability on thin fluid layers of a binary mixture

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Surface instabilities and structures in fluid thin layers are studied using the lubrication approximation [1]. The starting point is the governing equation for the film thickness that includes surface tension gradients ($\partial_x \sigma$), buoyancy forces ($\rho g h$) and Van der Waals forces ($\Pi(h)$) between substrate and free surface:

$$\partial_t h + \frac{1}{\mu}\nabla_2 \left\{ \frac{h^2}{2}\left(\nabla_2 \sigma\right) - \frac{h^3}{3}\nabla_2\left[\rho g h - \sigma \Delta_2 h - \Pi(h)\right]\right\} = 0 \ . \tag{1}$$

Here $\nabla_2$ and $\Delta_2$ are the operators in horizontal direction. The system is extended to a binary mixture if one assumes the surface tension as a (linear) function on both temperature and concentration $\nabla_2 \sigma = -\sigma_\theta \nabla_2 \theta - \sigma_n \nabla_2 n$.

Temperature and relative concentration are determined by their transport equations:

$$\rho c \left(\partial_t \theta + \overrightarrow{u} \cdot \nabla_2 \theta + w \partial_z \theta\right) = k_{th} \left(\Delta_2 + \partial^2_{zz}\right) \theta \tag{2}$$

$$\partial_t n + \overrightarrow{u} \cdot \nabla_2 n + w \partial_z n = D \left[\left(\Delta_2 + \partial^2_{zz}\right) n - \frac{\beta_n}{\beta}\left(\Delta_2 + \partial^2_{zz}\right)\theta\right] \tag{3}$$

with the thermal and mass diffusivity ($\kappa_{th}$, $D$), and the heat and concentration gradients $\beta$ and $\beta_n$. Here $\overrightarrow{u}$ and $w$ are the velocity components in the horizontal and vertical directions, respectively. Note that concentration gradients couple to temperature by the Soret-effect.

Oscillatory and monotonic long wave Marangoni instability in a binary-liquid layer with deformable interface in the presence of Soret effect was studied using two simplified sets of equations [2,3]. The vertical dependency of the temperature and concentration was approximated with polynomials and the convective term in equations (2) and (3) was neglected. Comparing the linear stability diagrams for the simplified and complete models (see Fig. 1) one can observe that the both

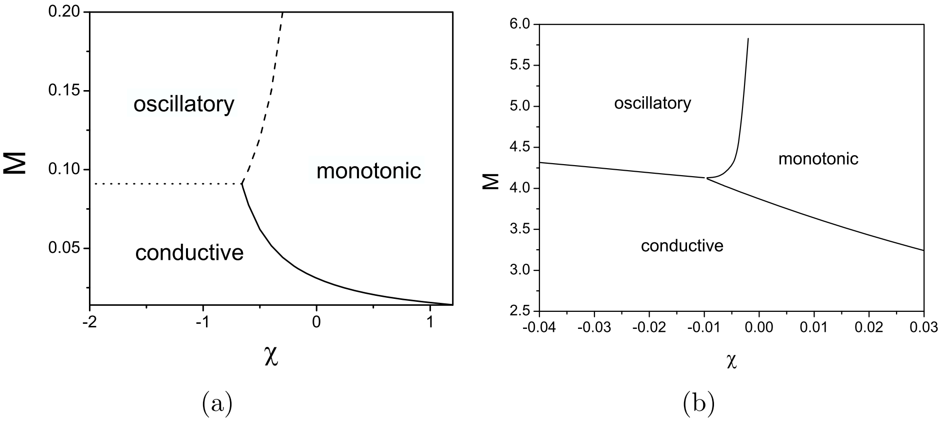


Figure 1: Linear-stability diagrams for the models express by the simplified equation system (left) [3] and using the 3D set of equation (right).

conductive and oscillatory instabilities are obtained, but for different Marangoni number values. One needs higher Marangoni number in the case of complete 3D model in order to obtain instabilities.

Using the simplified model we have studied the mechanism of the oscillatory instability induced by the Soret effect. For small Soret numbers the film height (gray scale) and the concentration field (contour lines) are superposed in the final stationary state (Fig. 2b). In the case of the oscillatory instability (bigger Soret number) the movement of the drop is due to the concentration difference between the two sides of the drop (Fig. 2c). The next step is to compare the non-linear behavior in the complete and simplified model and to deeply clarify the influence of the Soret effect on the oscillatory instability.

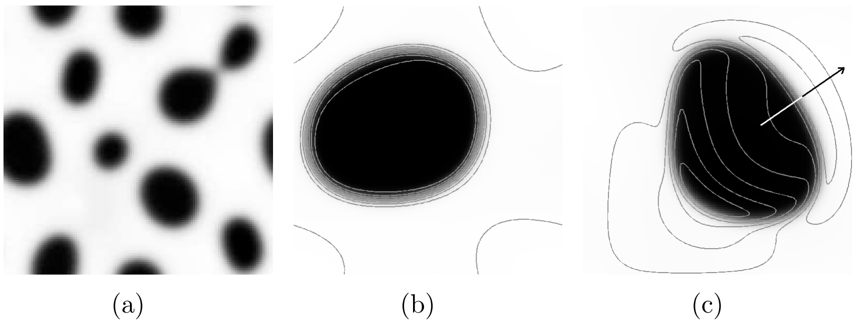


Figure 2: Nonlinear simulations using the simplified model for different Soret numbers: a) incipient development of the drops (calculated for $\chi = -0.8$, similar for $\chi = -1.2$), b) stationary final drop for $\chi = -0.8$ and c) traveling final drop for $\chi = -1.2$.

[1] A. Oron, S.H. Davis, S.C. Bankoff, *Rev. Mod. Phys.*, **69**(3), 931 (1997).

[2] I.D. Borcia, R. Borcia, M. Bestehorn, *Europhys. Lett.*, **75**(1), 112 (2006).

[3] I.D. Borcia, R. Borcia, M. Bestehorn, *J. Optoelectr. Adv. Mater.*, **8**(3), 1033 (2006).