Non-isothermal thin-film flow on a stationary or rotating cylinder

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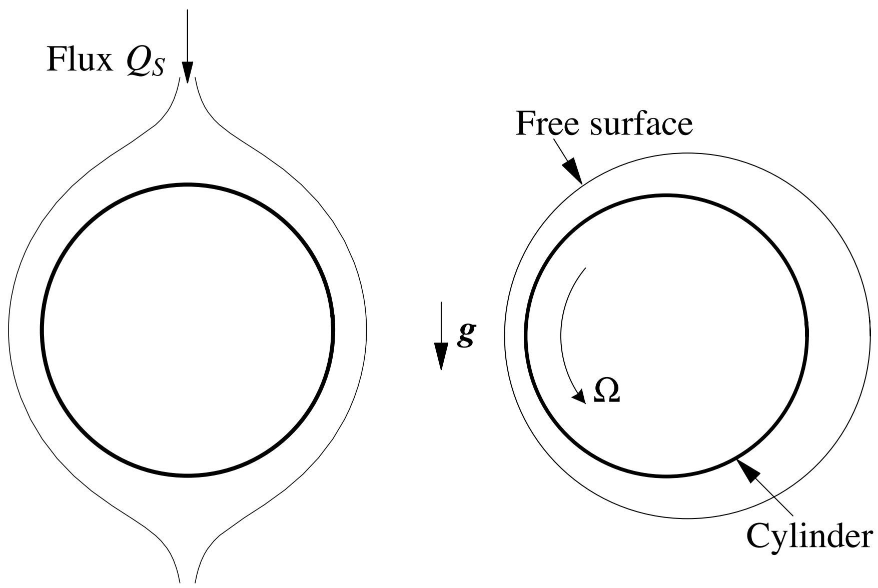
Problems concerning the flow of a film of viscous fluid on the inside or outside of a stationary or rotating horizontal cylinder are of relevance to, for example, condensers and coating processes in industry, but also they have come to be regarded as paradigm free-surface problems in the mechanics of viscous fluids, throwing up a variety of interesting phenomena such as shock formation and complicated three-dimensional instabilities (see, for example, Moffatt 1977, Johnson 1988, Ashmore *et al.* 2003, Acrivos & Jin 2004, and Evans *et al.* 2004).

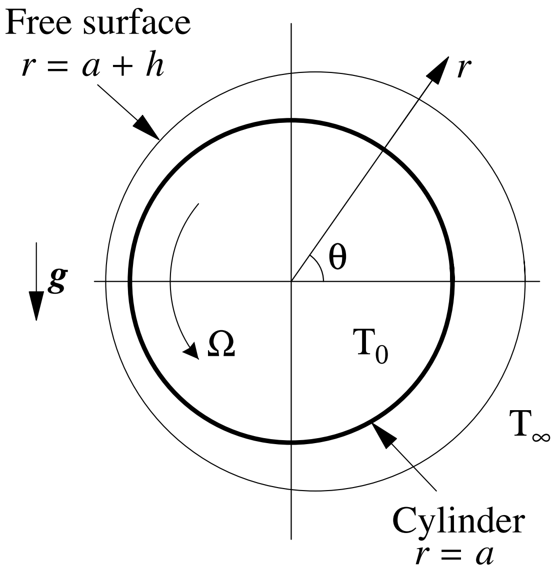
We use the lubrication approximation to investigate steady two-dimensional flow of a thin film of Newtonian fluid of uniform density ρ and temperature-dependent viscosity μ on a circular cylinder of radius a with its axis horizontal, the cylinder being at a uniform temperature T_0, hotter or colder than the (uniform) temperature T_∞ of the surrounding atmosphere. We consider both flow of a prescribed volume flux on the outside of a stationary cylinder (which we will refer to as "Nusselt flow"; cf Nusselt 1916*a*,*b*), and flow of a prescribed mass of fluid on a cylinder that is rotating about its axis at uniform angular speed Ω (which we will refer to as "Moffatt flow"; cf Moffatt 1977); in the latter case the fluid may be on the inside of the cylinder ("rimming flow") or the outside ("coating flow").



"Nusselt flow" "Moffatt flow"

Flow configurations.

We solve the leading-order thin-film versions of the mass-conservation, Navier–Stokes and energy equations subject to continuity of velocity and temperature on the cylinder,



Steady two-dimensional flow of a thin film of viscous fluid on a heated or cooled cylinder.

and stress balances, the kinematic condition and Newton's law of cooling at the free surface, to obtain the velocity, pressure and temperature of the fluid in terms of the film thickness $h = h(\theta)$. Then h is determined by prescribing the flux (Nusselt flow) or the mass of fluid (Moffatt flow).

In each type of flow we find that decreasing the ambient temperature has the effect of making the film thicker, and can lead to plug-like flow except in a region of strong shear near the cylinder. For coating flow we determine the maximum fluid load that can be supported on the cylinder; in particular, the maximum load increases with decreasing ambient temperature. Also we derive the correction to the isothermal solution in the limit of small Biot number, and we show that the film thickness (but not the fluid velocity) at large Biot number can be obtained from that in the isothermal case by a simple re-scaling.

References

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