Droplet Spreading: Quantitative Comparisons with Experiment

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Background and Motivation

Droplet spreading and wetting phenomena are ubiquitous throughout science and engineering and are crucial to several natural, engineering and manufacturing processes. Important examples arise in oil recovery [1], pesticide deposition on leaves [2], lab-on-a-chip devices and ink-jet printing [3]. Important analytical investigations apart [4], numerical solutions, based on lubrication theory coupled and a disjoining pressure term to alleviate the stress singularity at dynamic wetting lines, have increasingly appeared which explore the spreading motion of droplets: (i) on chemically- and topographically-heterogeneous substrates [5,6]; (ii) driven by external body or Marangoni forces [7,8].

Despite the success achieved computationally it is known that mesh resolution and precursor film thickness have a strong influence on predicted wetting speeds – too large a mesh spacing/precursor film thickness leads to significant over-predictions of the wetting speed – no systematic investigation into these effects or quantitative comparison with experimental data has been performed to date. The need for such is further highlighted in a recent comparison of numerical solutions [7] with the experiments of Podgorski et al [9] and LeGrand et al [10] for droplet spreading down an inclined plane, see Fig. 1. Although good qualitative agreement can be obtained without highly resolved grids a recent study by Koh [11] has shown that, far greater grid densities, with up to 250 million grid points, are required to obtain grid- and precursor film thickness independent solutions even for relatively simple droplet spreading flows.

The thrust, therefore, of this paper is to combine recent advances in the development of highly efficient Multigrid methods for solving the associated lubrication equations used to model droplet spreading with parallel computing architectures in order to explore such flows with physically realistic precursor film thicknesses [12] for the first time, and to compare the results with corresponding experimental data available in the literature. It is shown that, at the mesh densities required, wetting speeds can be predicted with an error that is less than 20% of what is observed experimentally – a significant and major advance over what has ever been achieved before. The methodology is underpinned by the recent advances made by the authors in the development of automatic adaptive mesh strategies devised for continuous film flows [13], which have been shown to result in considerable efficiency gains with respect to the use of computing resource without loss of accuracy.

The results of a complementary investigation, using the above methodologies, into the migration of a droplet up an inclined plane under the action of a surface energy gradient will be presented also, and comparison made with the experimental data of Chaudhury and Whitesides [14].

Sample Results

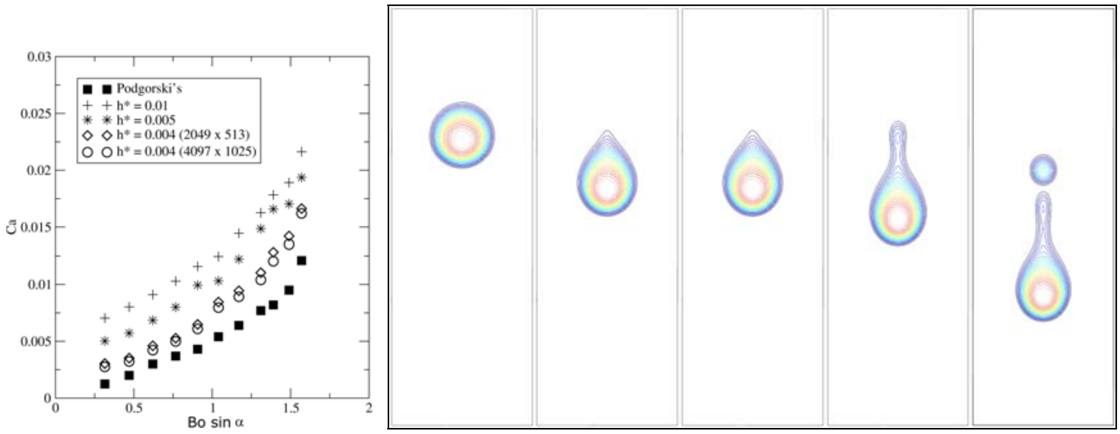

Figure 1: Quantitative comparison between experimental predictions [9] (left) and numerical simulations (right) for a 8.3 mm^3 47V10 silicon oil drop, spreading on an inclined plane, with advancing and receding contact angles, $\theta_A = 50^o$ and $\theta_R = 40^o$, respectively. The plot on the left shows the accuracy of numerical solution improves with smaller pre-cursor film thickness, h*, and the contour plots on the right illustrating the formation of droplets and pearlings with varying velocities, controlled by the inclination angle of the plane, at $20^o, 35^o, 40^o, 45^o$ and 50^o.

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