Meandering instability on a partially wetting surface

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Abstract

When a rivulet flows down an inclined plate, a meandering instability develops above a critical flow rate. We have performed experiments with water sliding down an inclined Mylar plate and determined the thresholds separating the different rivulet regimes: straight rivulets, stationary meanders, and dynamic rivulets. Dye visualizations have revealed interesting characteristics on the fluid velocity (dead zones, existence of reverse flow, etc…). At the transition straight/meandering, the mean velocity of the flow was found to drop drastically. The shape (amplitude, wavelength and radius of curvature) of the stationary meanders and its dependence on flow rate has been extracted from the experiments for increasing flow rates. Both the meandering threshold observed and the curvature radius of the meanders are interpreted by a simple model combining centrifugal forces, surface tension effects and wetting hysteresis. The rivulet flow is also shown to be highly hysteretic since the shape of the meanders remains unchanged when one decreases flow rate, and this until the rivulet eventually breaks into drops. Consequently, the straight rivulet regime can only be obtained for increasing flow rates.

Introduction

Meanders are a quite familiar pattern that everyone can see, either on a window during a rainfall or when a river meanders. Geologists have long been interested in meanders, trying to understand the serpentine paths that rivers dig in the soil [1], a mechanism in which soil erosion and sedimentation are involved. This pattern is in fact more general: recently, a remarkably regular meandering pattern has also been observed in a solution of surfactants, confined in a Hele-Shaw cell [2], and supposed to mimic high flow rate flows inside foam Plateau borders. The mechanisms at work in the three examples are certainly very different: there is no need of an erosion/deposition mechanism, nor of surfactants to get meanders, as these can be seen simply with water sliding down a solid plane. What is puzzling here is that, just as for a stream falling from a faucet, the "liquid particles" are pulled downwards by gravity, and it is difficult to understand why their trajectories should finally meander, as, in principle, the interaction with the substrate should limit possible lateral motions. So why do rivulets sometimes meander along an inclined plate? What conditions are needed for this instability to develop?

Rather few works on meandering down an inclined plane are available, especially when dealing with models [3, 4]. Most theoretical studies of rivulets are focused on the flow structure in straight rivulets [5], or on the varicose instability [6], and therefore fail to describe the meandering one (sinuose). Experimentally Nakagawa and Scott [7], and before them Culkin [8], have done an important work on meanders. They observed three regimes for increasing flow-rates: drops, meanders, and an unstable regime for which the main rivulet oscillates and splits into several smaller rivulets. They indicate that they do not get straight rivulets, though these have already been reported in other experimental works [9]. This point remains to be clarified, and as we will show later it is a matter of hysteresis in flow rate. Understanding meandering on a solid plate is important, for instance, in aeronautics. It has been shown that meander formation decreases aerodynamic efficiency of wings interacting with rain or clouds [10]. In heat exchangers also, the changes between straight rivulet and meandering can cause drastic modifications of heat transfer. These changes can also be undesirable in specific coating processes. Complete knowledge on the meandering instability (threshold, stability of the meanders, dependence on flow rate and plate inclination, etc…) would therefore be very helpful to optimize the efficiency of these industrial processes.

The present paper is organized as follows. A first section describes the experimental apparatus used to obtain and study the rivulets. A second section describes the rivulet regimes seen for increasing and decreasing flow-rate, and gives the conditions needed to achieve stationary meanders. A third section shows the variation of the mean velocity in the straight rivulet regime and at the transition to meanders. Finally, scaling laws giving the shape variation of the stationary meanders (amplitude, wavelength, radius of curvature) with flow-rate and plate inclination are indicated.

1. Experimental setup

A relatively simple apparatus is needed to obtain stationary rivulets (see figure 1). We inject de-ionized water (surface tension =72mN/m) at constant flow-rate, at the top of a solid plate of 1.20m long and 0.60m wide. The plate is covered in a Mylar sheet which creates partially wetting conditions for water, with advancing and receding contact angles of respectively $_a$=70° and $_r$=35°. Another advantage of using Mylar is that it creates very few problems of static electricity compared to most plastics. The plate can be tilted at will, denoting the angle between the plate and the horizontal. After flowing on the plate, the water is collected in a tank and a gear pump injects it back to the top of the plate. The constancy of the flow rate, ensured by the gear pump, is verified by means of a precision flow-meter.

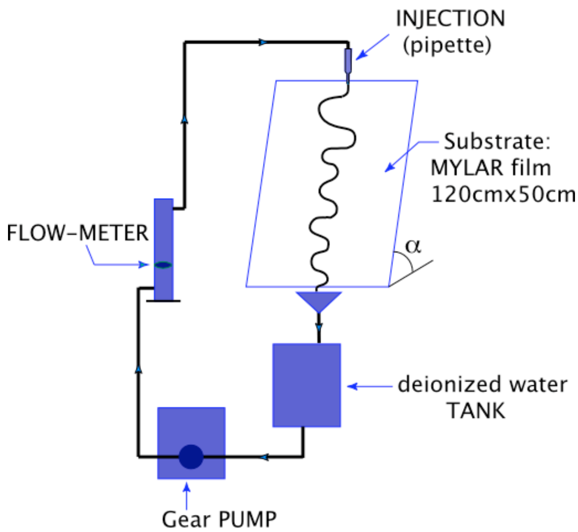


Figure 1: Schematic diagram of the experimental setup

Pictures and movies of the rivulets are taken with a digital camera placed 1m above the plate, perpendicular to it. The plate is lightened by reflection.

2. Rivulet regimes and thresholds

2.a Qualitative description of the rivulet regimes for increasing flow-rate

When one increases the flow rate, starting from zero, several regimes can be seen:

(i) the first one is a drop regime, which has already been widely studied [11].

(ii) For higher, but still very low flow-rates, one gets a rivulets regime which is `straight' (see figure 2*a*). We have called this regime `straight' since the lateral expansion of the stream remains of order of the width of the rivulet.

(iii) Above a first critical flow-rate, Q_{c1}, the straight rivulet is no longer stable and meanders develop (see figures 2*b* and *c*). What is interesting is that, after the laps of time needed for them to settle from top to bottom of the plate (it can take up to two hours for a rivulet to meander all over the 1.20m long plate), these meanders are completely stationary. Moreover, they are very stable: pictures from a settled stationary meander, which were taken every 2 minutes during 12 hours, showed that the meander did not move at all during that time laps. Another picture taken 24 hours later indicated that it still had not moved. This feature allowed us to study in details the shape of the meander. Figures 2*b* and *c* show two different shapes of meanders obtained for the same inclination but different flow-rates. The meander with the highest flow rate (figure 2*c*) is the wider and the larger. Indeed, as will be discussed in more details in section 4, the amplitude, wavelength and radius of curvature at the apex of the curves are all three increasing with flow rate, for a given inclination.

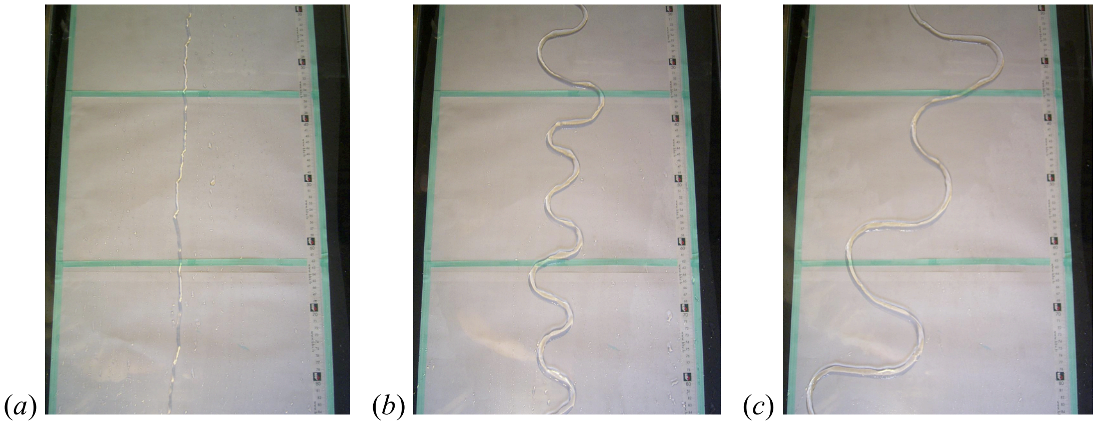


Figure 2: Water flows from top to bottom. Plate inclination: =6°.
(*a*) Straight rivulet for a flow rate Q=0.64mL/s (*b*) Stationary meander for Q=3.04mL/s
(*c*) Stationary meander for a higher flow rate Q=4.42mL/s.

(iv) If one now continues to increase the flow-rate and gets above a second critical value, Q_{c2}, the meanders can no longer remain stationary but act like a windscreen wiper, the stream of water constantly moving from left to right, spanning the width of the plate like a windscreen wiper and splitting into sub-rivulets. This last regime is called the `dynamic regime'.

2.b Threshold of the stationary meanders for increasing flow-rates; phase diagram

Figure 3 shows the thresholds of onset of stationary meandering (Q_{c1}) and of their end (Q_{c2}), for several plate inclinations. As explained above, the rivulet is straight before Q_{c1}, then meandering till Q_{c2}, and dynamic above Q_{c2}. The data giving Q_{c2} seem identical to the ones of Q_{c1} but simply shifted upwards. Curve fitting of the data giving the two thresholds gives power laws in sine of the inclination at the power –3/5 for both of them.

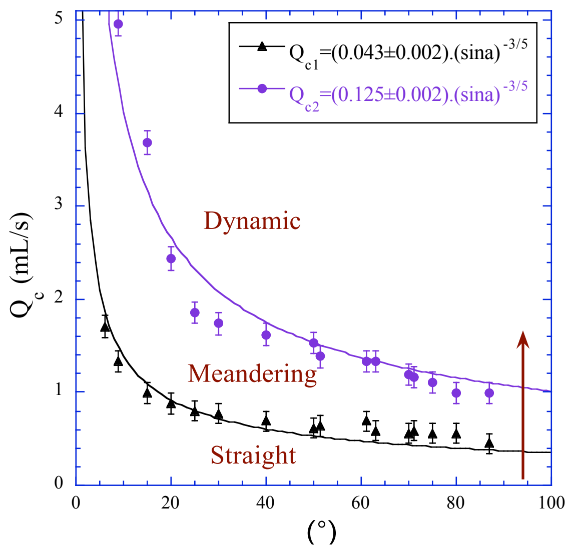


Figure 3: Phase diagram showing the separation between the different rivulet regimes

Let us try to explain these power laws by looking at what forces act on the rivulets. For the onset of meandering, we just concentrate on forces acting perpendicular to the rivulet. The forces at work are, per unit length:

- inertia: $S\frac{v^2}{R_c}$, where stands for the density of the liquid, S for the cross-section of the rivulet, R_c for its radius of curvature, and v for the velocity inside the rivulet
- line tension: $\frac{\ }{R_c}l$, where denotes the surface tension of the liquid and l the curve length of interface liquid/air
- hysteresis: $\ (\cos\ _r \quad \cos\ _a)$.

The equilibrium obtained is:

$$\frac{1}{R_c}(\ Sv^2 \qquad l) = \ (\cos\ _r \quad \cos\ _a). \qquad \text{(eq. 1)}$$

If $S\frac{v^2}{R_c} < \frac{\ }{R_c}l$ there is no solution to equation 1. The onset of meandering is given by the balance between inertia and line tension, as was also suggested by [3] for meanders in foams:

$$S\frac{v^2}{R_c} = \frac{\ l}{R_c} \qquad \text{(eq. 2)}$$

Assuming a Poiseuille flow, the velocity v inside the rivulet can be related to its width 2a:

$$v \quad \frac{a^2 g \sin}{3} \qquad \text{(eq. 3)}$$

Also assuming that the cross-section of the rivulet is nearly a half of a disc ($S \simeq \ \frac{a^2}{2}$ and $l = \ a$), the conservation of the flow-rate ($Q = Sv$) leads, by eliminating v and a, to:

$$Q_c \quad (\sin\ )^{3/5} \qquad \text{(eq. 4)}$$

which is the scaling law found experimentally.

2.c Decreasing flow-rate; hysteresis

When one proceeds at decreasing flow-rate, the behaviour of the rivulet is rather different from the one at increasing flow-rate. If a stationary meander (created at increasing flow rate) is settled on the plate and that one now decreases flow rate, the shape of the meander will not change at all until the flow rate becomes so low that the rivulet will eventually break into drops. Still, since the flow-rate is decreased, the meander gets thinner and thinner. This behaviour is totally different from what can be seen at increasing flow-rate, for which the shape varies with flow rate (see figures 2*b* and *c*, and section 4). Figure 4 shows hysteresis cycles that can be drawn for the mean amplitude <A> of the meanders, displaying its evolution when flow rate is increased or decreased. The same type of cycle could obviously be plotted for the wavelength < > and radius of curvature <R_c> of the meanders.

The cycle starts at zero flow-rate. When increased, the regime is straight until Q_{c1} and <A> is nearly zero. What must be noted on this figure is the jump occurring at the transition from straight to meandering. The same discontinuous transition can also be seen for < > and <R_c>. Once $Q>Q_{c1}$, there are stationary meanders, and <A> increases with Q. If, at any moment, the operator decides to decrease the flow-rate, then the shape of the meanders remains locked, <A> does not change anymore, and eventually, for a nearly zero flow-rate, the meander breaks into drops and a new hystersis cycle can be started.

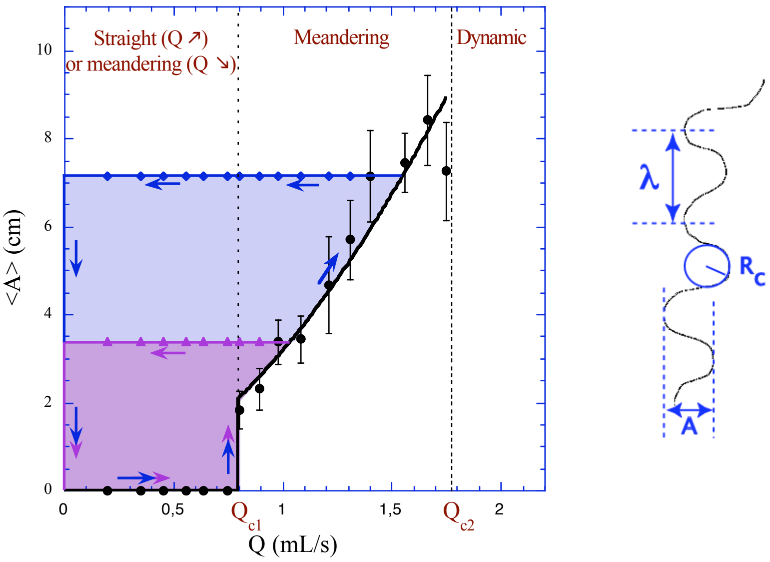


Figure 4: Hysteresis cycle on the amplitude, for =32°, along with the schematic description of the amplitude, wavelength and radius of curvature of the meanders

Another consequence of this hysteresis is that there is no straight rivulet whenever the flow-rate is decreased. This straight regime does only exist when the flow rate is increased, and this explains why it is sometimes mentioned, and sometimes not, in the literature. Therefore, the first critical flow rate Q_{c1} vanishes for decreasing flow rates. As for the second critical flow rate separating stationary meanders from the dynamic regime, it remains unchanged. Consequently, the phase diagram for decreasing flow-rates is identical to the increasing flow-rate one, except for the first critical flow-rate which does not exist anymore.

3. Velocity inside the rivulets

3.a Mean velocity

Injection of methylene blue dye in the rivulets, via a syringe, enabled us to visualize the velocity inside the straight rivulets and meanders. For straight rivulets, the velocity is constant all along the stream, and its value increases for increasing flow-rates (at a given inclination), as can be seen in figure 5.

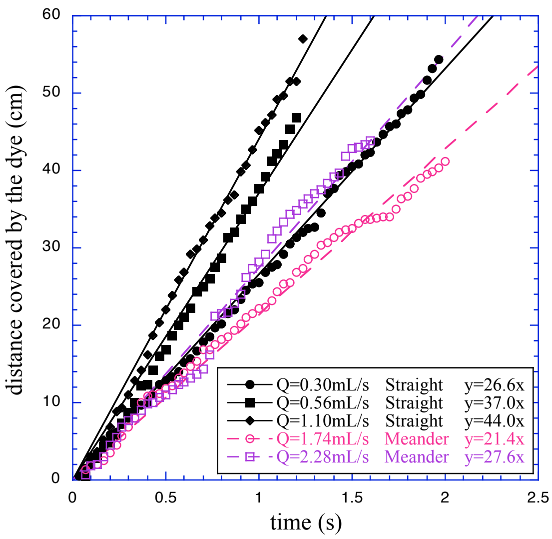


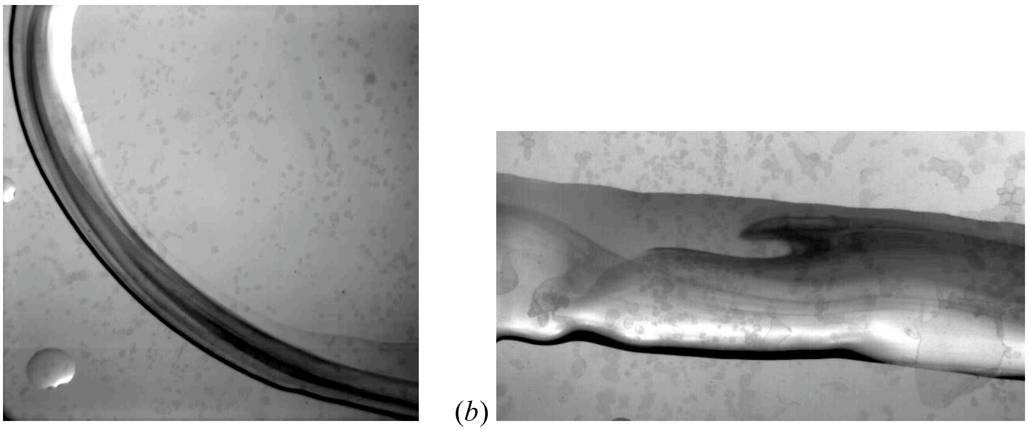
Figure 5: Evolution of the mean velocity inside straight rivulets (filled symbols) and meanders (open symbols). =9°. The velocity is given by the slope of the curves.

When one continues increasing the flow rate and gets at the transition from straight-to-meander, something quite surprising happens: the velocity drops considerably at the onset of meandering. The mean velocity then becomes as low as the lowest velocities for the straight rivulets. It seems that the rivulet prefers to avoid getting higher than a critical velocity and manages to do so by changing its shape in meanders. Once in meanders, the mean velocity continues to increase when the flow rate is increased (open symbols in figure 5).

It would be interesting to have more data on the velocity in order to get a closer value of the critical velocity, and to see if the second transition from stationary meanders to a dynamic regime occurs at the same critical velocity.

3.b Specific behaviour of the velocity field in meanders

Some interesting velocity fields have been observed in the stationary meanders regime. Dye visualizations revealed that small parts on the inner side of the meanders have a nearly zero velocity, and that the flow can even be reverse (see figure 6). The dye remains stuck in these `dead zones', whereas it is very rapidly evacuated in the rest of the meander (a typical value of the velocity there being 50cm/s), and eventually initiates a reverse whirling flow before being evacuated in its turn. Reverse flows in meanders have already been mentioned by Walker in an amateur column [12], but have never been explicitly shown.



(*a*) (*b*)

Figure 6: (*a*) `Dead zone' in the inner part of a meander =29° and Q=1,27mL/s. The dye flows from top to bottom (*b*) Reverse flow for the same meander. Main flow is from left to right, the dye creates a kind of wave breaking opposite to the main flow.

4. Quantitative description of the shape of the meanders

Since the shape of the meanders is unchanged when one decreases flow-rate, we will consider solely the case of increasing flow-rate for the following.

4.a Raw data

For the entire range in Q of the meandering regime, we have extracted from pictures the different values of the mean amplitude <A>, mean wavelength < >, and mean radius of curvature <R_c> of the meanders, for plate inclinations of =6°, 9°, 20°, 32°, 41°, 51°, 61°, 70°, 80°, and 87°. The corresponding results are given in figure 7.

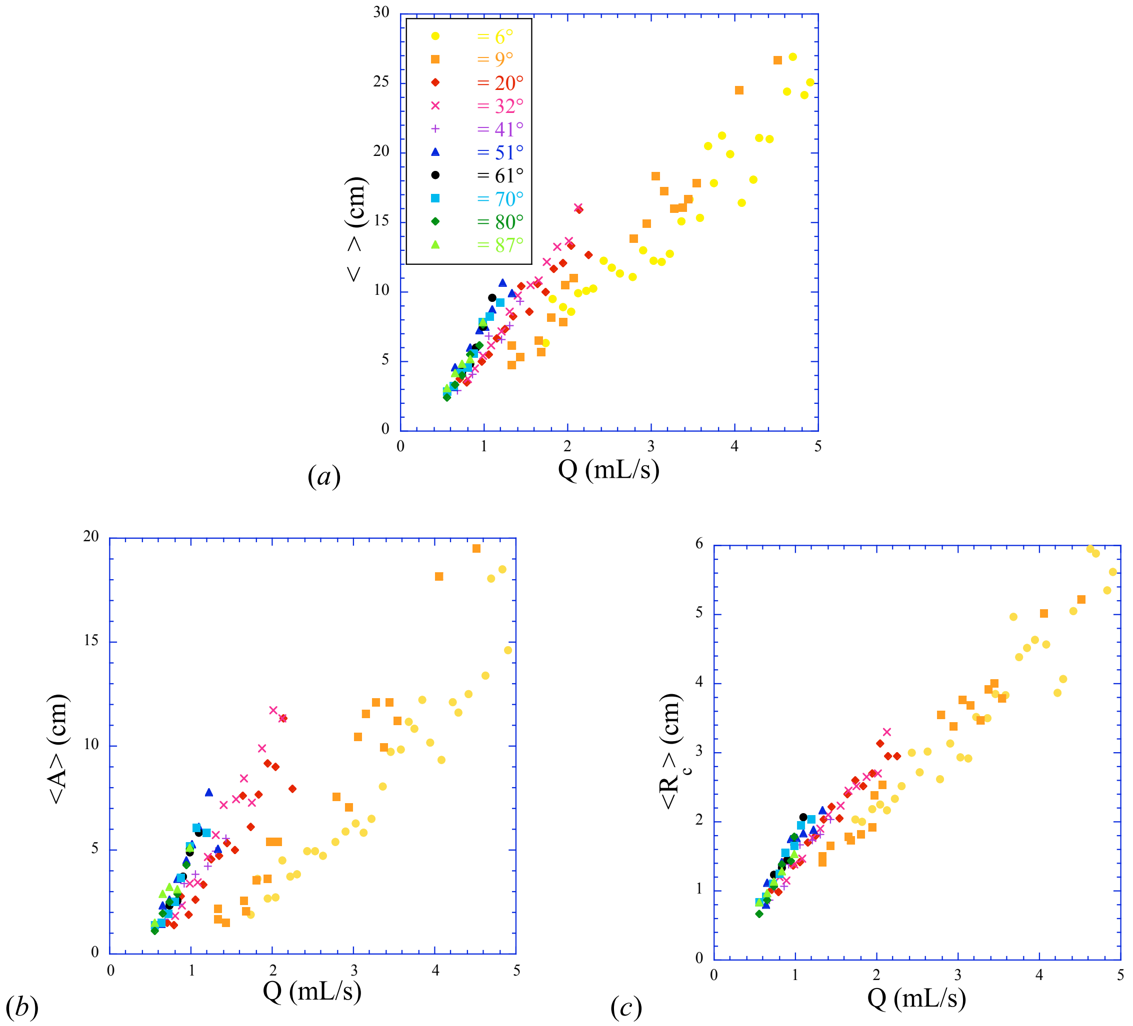


Figure 7: Raw data of the shape of the meanders. (*a*) mean wavelength (*b*) mean amplitude (*c*) mean radius of curvature

All three quantities increase with flow rate for a given inclination, reflecting the fact that the meanders become wider and the curves larger and larger. $<A>$, $< \ >$ and $<R_c>$ also increase with for a given flow-rate, i.e. increase when the plate gets more vertical. Let us now try to rescale all these data on one master curve.

4.b Scaling laws

As drawn in figure 8, when the rivulet is in stable meandering configuration, two opposite forces act on it: inertia and hysteresis of wetting. Indeed, for large flow rates at least, line tension is negligible compared to inertia.

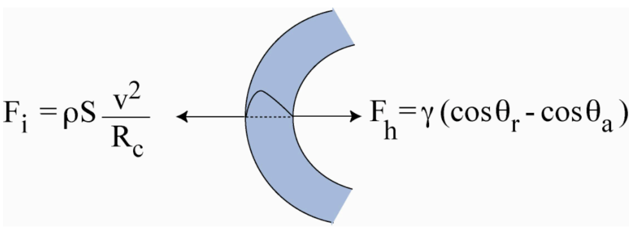


Figure 8: Forces acting on stationary meanders

The following balance is then obtained:

$$S\frac{v^2}{R_c} = \quad (\cos \quad_r \quad \cos \quad_a) \qquad \text{(eq. 5)}$$

Still assuming a Poiseuille flow, one gets the following expression for the radius of curvature:

$$< R_c >= \frac{}{(\cos \;_r \;\cos \;_a)}\sqrt{\frac{2g}{3}} \quad Q^{3/2}\sqrt{\sin \quad} \qquad \text{(eq. 6)}$$

We have tried to see if $< R_c > / \sqrt{\sin \quad}$ was a power law in Q, and the result is rather convincing (see figure 9).

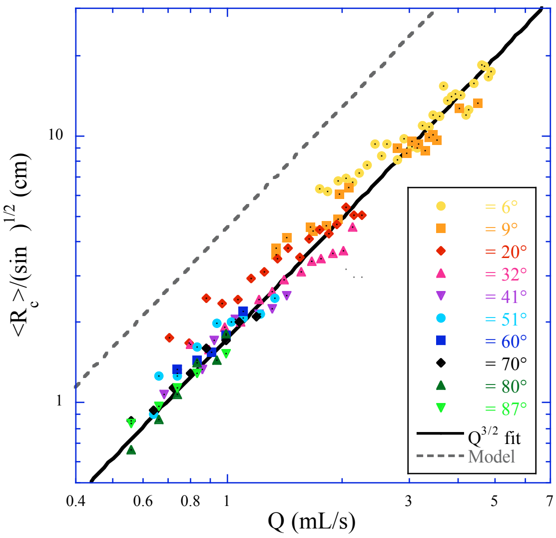


Figure 9: Data and model for the mean radius of curvature. The fit, in solid line, is a power law in $Q^{3/2}$ and the model, in dashed lines, is the complete formulae given in equation 6.

The scaling law (solid line) matches very well the experimental data, except perhaps that some sets of radii for given inclinations do not seem to have the same slope as the curve fitting in $Q^{3/2}$. Surprisingly, it even matches well for low flow rates, where the line tension should play a non-negligible role. The complete model with no tuneable parameter is plotted in dashed lines and is slightly above the experimental results. This could come from the fact that a Poiseuille flow was assumed whereas we have seen in section 3*b* that the velocity inside the meanders seemed to be more complex. Nevertheless, the model obtained is not so far from reality but could probably be improved by refining the modelling of the velocity.

Another issue that is not answered by this simple model is that it does not give any selection of wavelength. We have not got yet any modelling for the amplitude nor for the wavelength, but the experiments still enable us to find their scaling laws by using a least square method. The corresponding results are given in figure 10. The amplitude and the wavelength have been rescaled on a master curve, which shows that the wavelength and the amplitude can be scaled as:

$$< \quad > \quad Q^{4/3}(\sin \quad)^{1/3} \qquad \text{(eq. 7)}$$

$$< A > \quad Q^2(\sin \quad)^1 \qquad \text{(eq. 8)}$$

We now know how the shape of the stationary meanders evolves with flow-rate and plate inclination.

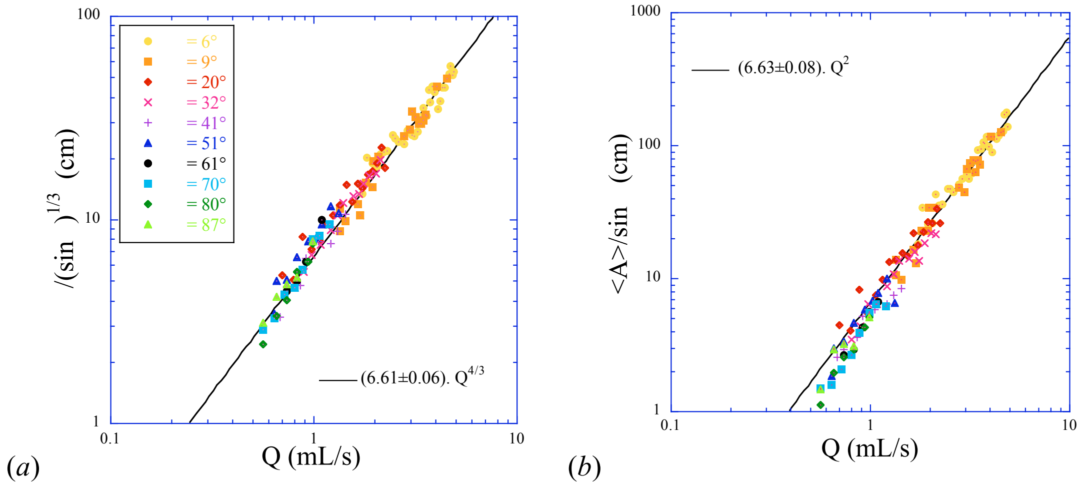


Figure 10: Rescaling of (*a*) the wavelength and (*b*) the amplitude, using a least square method.

The same least square method can also be applied to the radius of curvature. This method gives a scaling law in

$$\langle R_c \rangle \quad Q^{1}(\sin \quad)^{0.2}, \qquad \text{(eq. 9)}$$

as shown in figure 11, which is different from the one given by the balance between inertia and hysteresis of wetting (see equation 6 and figure 9), but stable meanders can only be achieved on a small range of flow-rate.

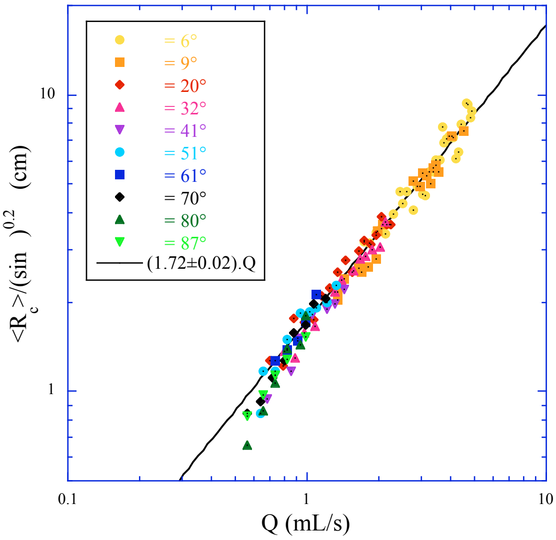


Figure 11: Least square method applied to the radius of curvature at the apex of the curves.

The exponents of the inertia/hysteresis scaling law can be lowered, which would bring them closer to the least square fit, by taking into account the sinuosity of the rivulet, which had been neglected until then. Indeed, the meanders are not straight, and the sinuosity $\mathcal{S}$, ratio of the total length of the meander to its projection on the line of greatest slope, is another characteristic of the shape of the meanders. The slope, and so the effect of gravity on the stream, is lower when the rivulet meanders than when it is straight. The corresponding effect is just a change from gsin   into g(sin   )/$\mathcal{S}$, which changes the power law in equation 6 into

$$\langle R_c \rangle \quad Q^{3/2}\sqrt{\frac{\sin}{\mathcal{S}}}\,. \qquad \text{(eq. 10)}$$

Since the sinuosity increases both with the inclination of the plate and with the flow rate [7], this correction to the model decreases the exponents of both Q and sin , which would come closer to the least square fit. A detailed study of the variation of the sinuosity with flow rate and plate inclination would allow us to compare the inertia/hysteresis modelling to the least square method quantitatively.

Conclusion

We have built an experimental setup enabling us to obtain meanders that are totally stationary and that can remain stable for many hours, and so that can be easily studied. We have found experimentally and also modelled the laws giving the evolution of the two critical flow rates limiting the stationary meandering regime. Also, the straight rivulet regime only exists when the experiment is done by increasing flow-rate. If flow-rate is decreased, meanders keep their shape until the rivulet breaks-up into drops. No straight rivulet can be seen for decreasing flow-rates, which reveals a very important hysteresis. Therefore, the lower critical flow rate is non-zero only for increasing flow rates but vanishes when flow-rate is decreased. We now know under what conditions the regime changes occur, which could be useful for some industrial processes, but have also found how the shape of the meanders changed.

The amplitude, wavelength and radius of curvature of the meanders increase with inclination of the plate and with flow-rate. The scaling laws giving their increase with these two parameters have been found. A simple model involving inertia and the hysteresis of wetting explains quite well the scaling law of the radius of curvature, but there still remains a little discrepancy between the complete model and the experimental data. A better modelling of the velocity than a Poiseuille flow would certainly improve the modelling. Anyway, the model fails to explain the existence of a selection in wavelength or amplitude. A more complete model involving partial differential equations, capturing the physics that we have identified and possible other effects, remains to be built.

Finally, dye visualizations showed that the velocity inside the rivulets dropped considerably at the transition from straight-to-meandering rivulet. Everything happens as if the rivulet did not want to continue increasing its velocity above a critical value, and by changing its shape into a meander, managed to go well below this value. We have also highlighted the particularities in the velocity field of stationary meanders in which `dead zones' with nearly zero velocity can be seen, as well as some whirling reverse flows. More data on the evolution of the velocity with the flow-rate and an improved model of the velocity in meanders would certainly help to understand more completely the meandering instability.

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