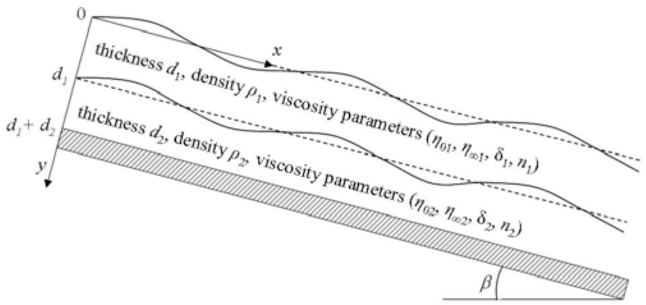
Stability analysis of stratified coating flow of shear-thinning fluids

Millet Séverine, Rousset François, Botton Valéry and Ben Hadid Hamda
Université de Lyon, Université Lyon 1, INSA de Lyon, Ecole Centrale de Lyon,
Laboratoire de Mécanique des Fluides et d'Acoustique UMR CNRS 5509
severine.millet@insa-lyon.fr

Simultaneous coating of multiple layers is required in many applications. For that reason processes such as slide coating and curtain coating are commonly used. One of the encountered defects of these processes is related to waves that may develop in the flow along the external surface of the die (called the slide) and results in non-uniform layer thickness. Indeed, the different liquid layers are stacked on the inclined surface of the die and the flow can amplify waves initiated by ambient perturbations that degrade the uniformity of a coating. Wave propagation can be examined through stability analysis of a stratified gravity-driven flow down an inclined. Most of the studies in the literature about this field are based on the Newtonian fluid model. However, many fluids used in coating flows exhibit a shear-dependant viscosity.

In the present study we focus on fluids following the four-parameter Carreau inelastic model. This law accurately describes the rheological behavior of fluids like suspensions of polymers and melt polymers which are known to be shear-thinning. Indeed for such fluids, an increase in the shear rate leads to a decrease in the fluid viscosity. The Carreau model predicts in particular a power-law behavior at moderate shear rate. However, unlike the power-law model, it predicts a viscosity η_0 that remains finite as the shear rate approaches zero. This feature makes the Carreau law particularly suitable to free surface flow issues. In the present case the temporal stability study leads to a modified Orr-Sommerfeld equation. A spectral Tau collocation method based on Chebyshev polynomials is used for the discretization of the generalized eigenvalue problem.

The case of superposed layers of Newtonian fluid films flowing down an inclined plane has been extensively studied over the years. Loewenherz and Lawrence [1], Chen [2] have studied the linear stability of a two-layer film flow with a free surface. They showed that when the viscosity of the upper layer is larger than that of the lower layer, the flow is unstable even without inertia. This interfacial instability can have two distinct wavelengths: a long wavelength, which is always unstable, and a moderate wavelength (on the order of layer thickness) strongly influenced by the flow parameters like the layers thickness ratio or the surface tensions. According to the flow parameters, one or the other among these two wavelengths exhibits the greatest growth rate and determines the instability wavelength. Hu *et al.*[3] identified in the inertialess case a threshold value of the density ratio, under which the moderate wavelength instability vanishes for any viscosity ratio.



Density ratio: $\dfrac{\rho_1}{\rho_2}$

Viscosity η_k in layer k : $\dfrac{\eta_k - \eta_{\infty k}}{\eta_{0k} - \eta_{\infty k}} = \left[1 + \left(\delta_k \left|\dot{\gamma}_k\right|\right)^2\right]^{\frac{n_k - 1}{2}}$

Definition sketch

In this study the effects of density stratification are studied in the case of a less viscous layer adjacent to the wall (favorable to interface instabilities). We highlight the existence of a threshold of density ratio from which is observed a switch from a long wavelength instability to a moderate wavelength instability. We also investigate the influence of the shear-thinning properties of the fluids on the position of this threshold. We show that, when one of the two layers is shear-thinning (Carreau law), this density ratio threshold depends on the position of this layer. If the lower layer is shear-thinning, the moderate wavelengths are destabilized, and the value of the threshold decreases. It means that the zones of moderate wavelength instabilities become larger in this case. On the other hand, when the shear-thinning fluid is in the upper layer, in comparison with the Newtonian case, moderate wavelength instabilities are stabilized and the switch is observed for higher values of density ratios. A stabilization of the long wavelength instability is shown whatever the position of the shear-thinning layer.

We also rely on an energy budget on the two-layer Carreau fluid flow problem to explain these differences on the stability according to the density stratification. We investigate the key role played by the shear stress at the undisturbed free surface to explain those phenomena and the influence of the shear-thinning properties on the different contributions of the energy balance.

[1] D. S. Loewenherz, C. J. Lawrence, The effect of viscosity stratification on the stability of a free surface flow at low Reynolds number. Phys. Fluids A. 1(10) (1989) 1686-1693.

[2] K. Chen, Wave formation in the gravity-driven low-Reynolds number flow of two liquid films down an inclined plane. Phys. Fluids A. 5(12) (1993) 3038-3048.

[3] J. Hu, S. Millet, V. Botton, H. BenHadid, D. Henry, Inertialess temporal and spatio-temporal stability analysis of the two-layer film flow with density stratification. Phys Fluids. 18(10) (2006) 104101.