A general free surface rule for Stokes flow of fluid films over obstacles

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Abstract

A general rule is derived for the free surface profile of a gravity–driven fluid film on an inclined wall as it flows over a local irregularity, or obstacle, in the present case a trench, see Fig. 1. Starting from an exact analytical solution of Stokes' equations, based on complex function theory, it is proved that the integral of the free surface profile, from its planer asymptotic equilibrium level, vanishes. Hence, in Fig. 1 the hatched areas above and below the dashed line are of equal size. This general analytical result is not only an interesting feature in itself but also provides a useful check of the accuracy of numerical schemes that are used to solve such problems.

For the validity of this surface rule only two requirements have to be fulfilled: the flow geometry must be two–dimensional and the Stokes' condition $Re_L \ll 1$ must be fulfilled where the corresponding Reynolds number

$$Re_L = \left(\frac{L}{L_i}\right)^3 \tag{1}$$

is the third power of the aspect ratio of the width of the topography L and the intrinsic length:

$$L_i := \left[\frac{2\nu^2}{g\sin\alpha}\right]^{1/3}, \tag{2}$$

where ν denotes the kinematical viscosity, g the gravity acceleration and α the inclination angle of the plane. Hence, $Re_L \ll 1$ means that the spatial extension L has to be small compared to an intrinsic length L_i, whereas the shape of the irregularity may be arbitrary.

To quantify the validity limit of the surface rule a parameter study is performed for a trench with rectangular shape, in which the weak form of both the Stokes and

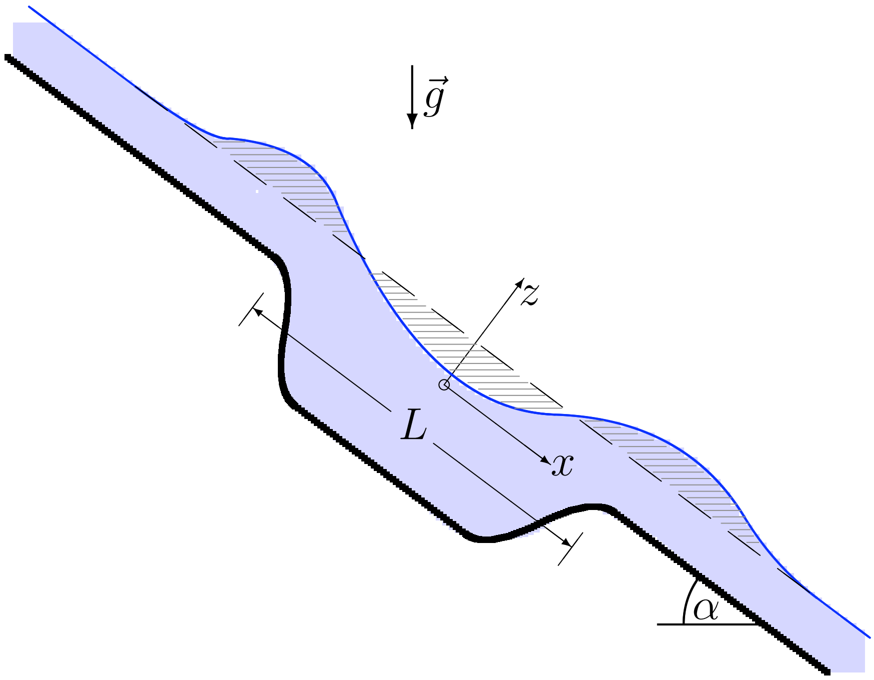

Figure 1: Sketch of a film flowing over a wall containing a localised trench irregularity.

Navier–Stokes equations for the two-dimensional free–surface flow are solved using the finite element method. The resulting integral of the free surface profile is calculated for increasing trench width L and therefore for increasing Re_L.