Abstract Submitted to 7th European Coating Symposium (ECS 2007),
Paris, France, 12th-14th September 2007



# Unidirectional flow of a thin rivulet of perfectly wetting fluid subject to a prescribed uniform longitudinal shear stress at its free surface

J. M. Sullivan, S. K. Wilson[1] and B. R. Duffy

Department of Mathematics, University of Strathclyde, Livingstone Tower,
26 Richmond Street, Glasgow G1 1XH, United Kingdom

There are many practically important situations in which an external airflow has a significant effect on the behaviour of a film of fluid, and consequently a considerable amount of theoretical and numerical work has been undertaken in order to understand the flows that can occur. One context in which these ideas may have practical application is the intriguing problem of the so-called Rain-Wind Induced Vibrations (RWIVs) of the cables of cable-stayed bridges. RWIV are caused by interaction between the wind and rivulets of rainwater on the cables and can be severe enough to cause significant damage to the bridge. Almost all of the previous work on rivulets concerns non-perfectly wetting fluid (i.e. fluid with a non-zero contact angle at its contact lines). There has been much less work on rivulets of perfectly wetting fluid (i.e. fluid with zero contact angle at its contact lines).

In the present work we use the lubrication approximation to obtain a complete description of the steady unidirectional flow of a thin rivulet of perfectly wetting fluid on a planar substrate inclined at an angle $\alpha$ ($\pi/2 < \alpha \leq \pi$) to the horizontal with semiwidth $a$, maximum thickness $h_{\mathrm{m}}$ and volume flux $\bar{Q}$ subject to a prescribed uniform longitudinal shear stress $\tau$ at its free surface. As well as being of interest in their own right, rivulet solutions for varying $\alpha$ can be interpreted as describing flow down a slowly varying substrate such as, for example, flow in the azimuthal direction round the lower part of a large horizontal cylinder. In particular, we obtain and classify all of the possible solutions and catagorise all of the possible cross-sectional flow patterns.

Having determined all of the possible rivulet solutions, the next step is to consider whether or not these rivulets can occur in practice.

Figure 1 shows how the $\tau$–$h_{\mathrm{m}}$ parameter plane is divided into stable (shaded) and unstable (unshaded) regions according to a quasi-steady stability analysis. Specifically, Figure 1 shows that for $\tau > 0$ rivulets with $h_{\mathrm{m}} \leq 3\tau/5 \sin\alpha$ are unstable and rivulets with $h_{\mathrm{m}} > 3\tau/5 \sin\alpha$ are stable. Figure 1 also shows that for $\tau \leq 0$ rivulets with $h_{\mathrm{m}} > h_{\mathrm{m0}}$, $h_{\mathrm{m}} = h_{\mathrm{m0}}$ and $h_{\mathrm{m}} < h_{\mathrm{m0}}$ are stable, neutrally stable and unstable, respectively, where $h_{\mathrm{m0}} = -9\tau/5 \sin\alpha$. Figure 1 also shows the solutions for $h_{\mathrm{m}}$ for $\bar{Q} = -5, \ldots, 5$ in order to indicate which are stable and which are unstable.

Figure 2 shows whether or not it is energetically favourable for a rivulet to split into subrivulets. In particular, Figure 2 shows the (shaded) region of the parameter plane bounded by $h_{\mathrm{m}} = h_{\mathrm{m0}} = -9\tau/5 \sin\alpha$ for $\tau < 0$ and the curve $\Delta E_2 = 0$ in which it is unfavourable for a rivulet to split, and that when $\tau \geq \tau_{\mathrm{c}}$, where $\tau_{\mathrm{c}} = (2/3)^{1/3} \simeq 0.8736$, and when $h_{\mathrm{m}} < h_{\mathrm{m0}}$ for $\tau < 0$ the most energetically favourable state is that with infinitely many subrivulets. The remainder of the parameter plane is divided by the critical curves $\Delta E_n = 0$ for $n = 3, 4, 5, \ldots$ (shown with dashed lines) into regions in which the state with $n$ subrivulets is energetically favourable, and by the critical curves $\Delta E_{n,n+1} = 0$ for $n = 2, 3, 4, \ldots$ (shown with solid lines) into regions in which the state with $n$ subrivulets is the most energetically favourable.

The first author (JMS) gratefully acknowledges the support of the Engineering and Physical Sciences Research Council via a studentship. All the authors gratefully acknowledge useful discussions with Dr Ian Taylor and his colleagues in the Department of Mechanical Engineering at the University of Strathclyde.

[1]Corresponding Author. Telephone : + 44 (0)141 548 3820, Fax : + 44 (0)141 552 8657, Email : s.k.wilson@strath.ac.uk

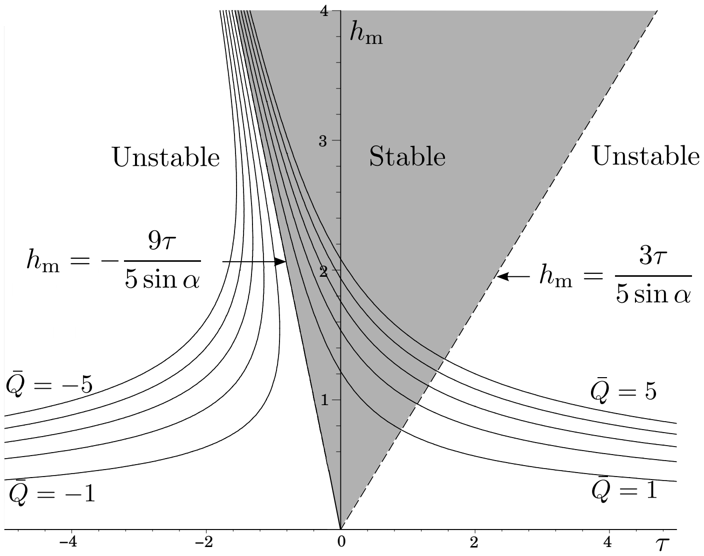


Figure 1: Plot of the $\tau$–$h_{\mathrm{m}}$ parameter plane divided into stable (shaded) and unstable (unshaded) regions according to the present quasi-steady stability analysis.

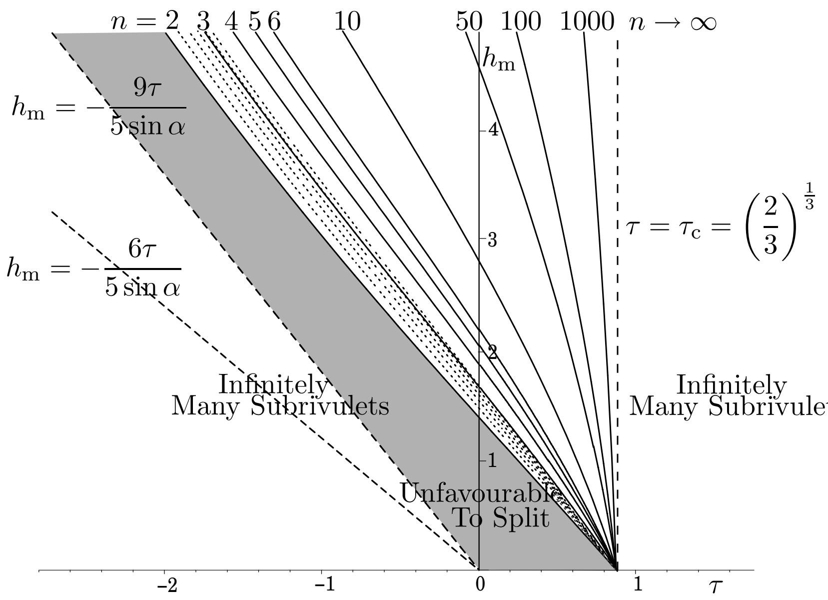


Figure 2: Plot of the $\tau$–$h_{\mathrm{m}}$ parameter plane showing the (shaded) region bounded by $h_{\mathrm{m}} = h_{\mathrm{m0}} = -9\tau/5\sin\alpha$ for $\tau < 0$ and the curve $\Delta E_2 = 0$ in which it is unfavourable for a rivulet to split, and that when $\tau \geq \tau_{\mathrm{c}} = (2/3)^{1/3} \simeq 0.8736$ and when $h_{\mathrm{m}} < h_{\mathrm{m0}}$ (including the curve $h_{\mathrm{m}} = -6\tau/5\sin\alpha$) for $\tau < 0$ the most energetically favourable state is that with infinitely many subrivulets.