Capillary instabilities of rivulet coatings

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In addition to the long standing interest in thin films of liquid inside wedges as a result of capillary imbibition, there is recent interest in coating of liquids in rivulets for microfluidics applications. This article presents the study of capillary instabilities of liquids in wedges, a rivulet on a horizontal surface being a special case. We are interested in the nonlinear evolution process of breakup. As is well known [1], when a free cylinder of fluid is perturbed, surface tension induces a capillary pressure gradient which enhances the perturbation and causes instabilities. Potential for capillary instability exists for rivulets as well, provided the rest state is convex. However, in the rivulet case, contact lines are present and their dynamics couple to that of the fluid in a significant way.

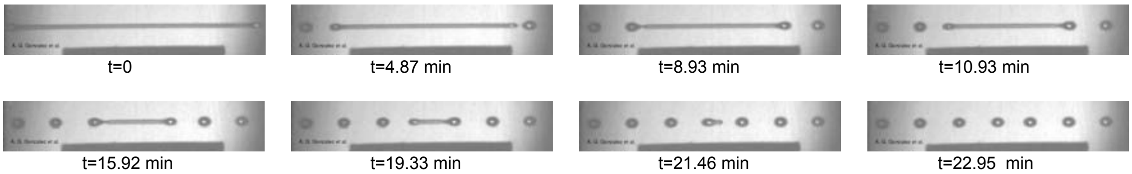


Figure 1. Photographs of a pearling process of a fluid strip on a partially wetting substrate [2].

A recent experiment by Gonzalez *et al.* [2] investigated the evolution of a long strip of viscous fluid (silicon oil) sitting on a horizontal glass substrate under partial wetting conditions. Due to a local perturbation at both ends of the finite rivulet, a breakup process propagates from the extremes toward the strip center, as shown in Fig. 1. Eventually the liquid strip breaks into an array of droplets involving primary, secondary, tertiary and even smaller droplets.

Although there are analyses of the linear stability of rivulets [3,4,5], as far as we know, there is no nonlinear theory of such capillary instabilities with contact lines. Our main objectives of this work are to formulate the nonlinear problem, to explore the evolution process numerically, and to the extent possible, compare with the recent experimental results.

We consider a liquid meniscus inside a wedge of included angle 2β that wets the solid walls with a contact angle θ. The meniscus has a convex interface which satisfies $\pi/2 < \theta + \beta < \pi$. The quantity a(z,t) measures the distance from the wedge corner to the contact line formed by the intersection of the interface and the side wall. Of course the location of the contact line is not known in advance, which leads to a free boundary problem. As explained in [3], any dynamic contact angle model must be analytic in the speed. We adopt a model in which the instantaneous contact angle depends on the contact-line speed as

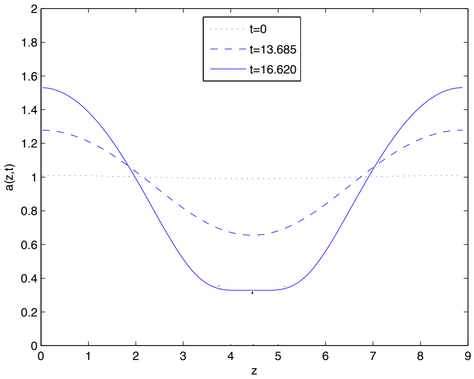
$$\theta = \theta_0 + G a_t \, .$$

Here θ_0 is the static contact angle and G is a constant of the model. The special case of G=0 corresponds to a fixed-contact-angle and perfect slip condition. This dynamic contact line model together with a thin film equation whose derivation is given in the full paper, constitute a nonlinear system for the unknowns a(z,t) and θ. The thin film equation has a clear physical interpretation: the cross section area changes in time due to the liquid flow driven by the capillary pressure gradient which in turn involves both transverse and axial curvature effects [6]. With scales derived from the linear stability theory, the dimensionless nonlinear system takes the following form:

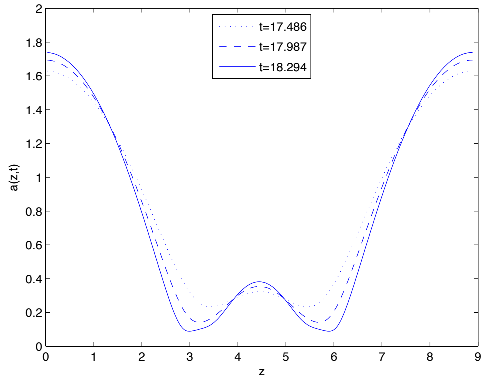
$$a_t = \frac{\theta - \theta_0}{G} \, ,$$

$$\theta_t = -\frac{2E}{Fa^2}\frac{\theta-\theta_0}{G}a + \frac{2E_0}{Fa^2}\left(\frac{Q^*}{Q_0^*}a^4\left(\frac{k/k_0}{a} - \frac{A_1}{A_{2,0}}\frac{a_z^{\,2}}{a} - \frac{A_2}{A_{2,0}}a_{zz}\right)_z\right)_z \, .$$

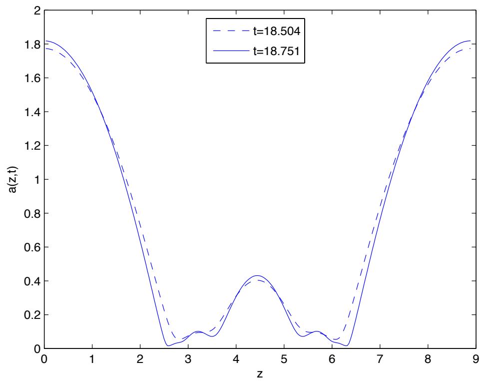
22 Here A_1, A_2, E, F and Q^* are known functions of θ and β; $A_{2,0}$, E_0, F_0 and Q^*_0 are values of A_2, E, F and Q^* for $\theta=\theta_0$. The non-linear system is solved numerically for axially periodic boundary conditions using spectral methods.



(a)



(b)



(c)

Figure 2. Profiles of dimensionless a(z,t) for $\theta_0=\pi/4$, $\beta=\pi/2$ and G=0 at different times. (a) Primary instability; (b) Secondary instability; (c) Tertiary instability and suggestion of a fourth stage.

Figs. 2(a)-(c) give typical results, the curves showing the shape of the contact line for different times. The evolution occurs in three distinct regimes. Fig. 2(a) demonstrates the growth of the primary instability into the nonlinear regime. Notably, the liquid gathers into droplets while the central of the contact line becomes flat as shown for t=16.62. Fig. 2(b) shows the continuation of the evolution leading to a secondary instability: for $16.62<t<18.3$, a bulge is formed in the center of the flat region and two secondary necks appear symmetrically beside the bulge. The further evolution is shown in Fig. 2(c): the secondary neck becomes flattened and a tertiary instability is initiated, generating two secondary bulges. The tertiary instability continues until a fourth instability stage originates, as suggested by the slight flattening and slight bulging near z=2.8, 6.2 in Fig. 2(c) for t=18.751. Thus capillary instability leads to a cascade of primary, secondary, tertiary, and presumably even smaller droplets, in qualitative agreement with similar phenomena observed in the experiment [2].

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