The Flow of Thin Liquid Layers on Circular Cylinders

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The time evolution of a coating layer of liquid on an inclined circular cylindrical substrate is studied experimentally and theoretically. For small-diameter cylinders, the motion of Newtonian liquids, driven by the combined influences of surface tension and gravity, is analyzed using the long-wave or "lubrication" approximation. Computed time-dependent solutions of the lubrication model are in general agreement with our experimental video observations. Starting from a slightly-perturbed uniform coating, the full family of evolving flows is shown to depend only on three dimensionless parameters: the inclination angle of the cylinder from the vertical direction, the Bond number representing the ratio of gravity to surface tension effects, and a nondimensional measure of the initial coating thickness. Typically flow is initiated by the well-known Rayleigh instability followed by drainage, and also wave propagation if the cylinder is not horizontal. Steady propagation of ring-like structures can occur as well as eventual overtaking, merging and reformation of the rings. We demonstrate that volumetric transport is maximized if the cylinder axis is inclined to, rather than aligned with, the direction of gravity. Results are relevant to the understanding, and potential optimization, of small-scale liquid transport. Such problems arise in the natural and industrial worlds.

The lubrication or quasi-parallel flow assumption often allows the continuity and momentum equations to be combined into a single evolution equation. An approximate equation for the coating thickness $h(x, \phi, t)$ is

$$h_t = -\nabla \cdot \mathbf{Q} = -\frac{\sigma}{\mathbf{3}\mu}\nabla \cdot \left[\mathbf{h^3} \quad \nabla\nabla^2\mathbf{h} + \frac{\nabla\mathbf{h}}{\mathbf{R^2}}\right)\right] - \frac{\rho\mathbf{g}\cos\alpha}{\mathbf{3}\mu}\frac{\partial\mathbf{h^3}}{\partial\mathbf{x}} - \frac{\rho\mathbf{g}\sin\alpha}{\mathbf{3}\mu}\frac{\partial}{\partial\mathbf{y}}\left[\mathbf{h^3}\sin\left(\frac{\mathbf{y}}{\mathbf{R}}\right)\right]$$

$$+\frac{\rho g\sin\alpha}{3\mu}\nabla\cdot\left[h^3\nabla\left(h\cos\left(\frac{y}{R}\right)\right)\right] \; .$$

The nabla is a two-dimensional operator in the substrate coordinates x and y where $y = R\phi$. The equation is nondimensionalized using h_0 as an initial or reference thickness for h and R as the reference quantity for x and y. The characteristic time is $T^* = 3\mu R^4/(h_0^3\sigma)$. The resulting

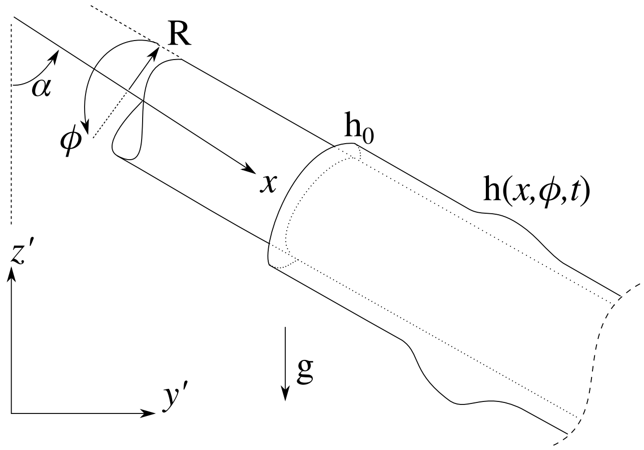


Figure 1: Definition sketch showing a cylinder whose axis is inclined from vertical at angle α. The cylinder radius is R and the coating thickness is given by $h(x, \phi, t)$.

dimensionless equation is

$$h_t = -\nabla\cdot\left[h^3\left(\nabla\nabla^2 h + \nabla h\right)\right] - \frac{B}{H}\cos\alpha\,\frac{\partial h^3}{\partial x} - \frac{B}{H}\sin\alpha\frac{\partial(h^3\sin y)}{\partial y} + B\sin\alpha\;\nabla\cdot\left[h^3\nabla\left(h\cos y\right)\right]\ .$$

There are three dimensionless parameters: the inclination angle α, the initial coating thickness ratio $H = h_0/R$ and the Bond number $B = \rho g R^2/\sigma$.

There are five terms on the right side of the evolution equation. In order, from left to right, they are (i) the stabilizing surface tension term,

(ii) the destabilizing term which drives the Rayleigh or sausage instability

(iii) the gravity-driven flow component in the axial direction,

(iv) circumferential gravity drainage which will lead to an accumulation of liquid on the underside of the rod,

and (v) the gravity component that is normal to the cylinder axis. This term is stabilizing on the upper side where $\cos y > 0$ and destabilizing on the underside of the inclined rod where liquid is accumulating.

Thin fibers were withdrawn at a slow steady speed from from a viscous liquid bath. The sensible coating thickness was controlled by the withdrawal speed according to the Landau-Levich law. An experimental test fixture was constructed to perform consistent and steady motions to withdraw and orient the rod. Depicted in Figure 2, the fixture consists of a vertical gantry, driven by a stepper motor via a timing belt, to move a stage a prescribed distance and velocity in the vertical direction. Mounted to the stage, a servo motor actuates a 4-bar mechanism, on which the rod is mounted. The servomotor moves to a finite angle, moving the rod in the y-z plane into the frame of a camera.

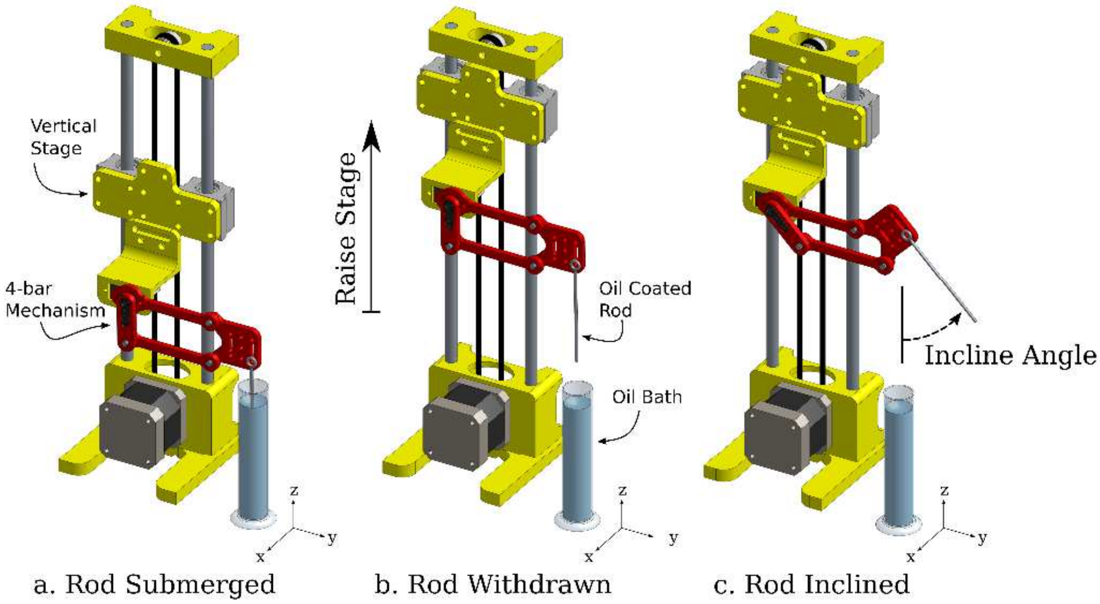


Figure 2: Experimental test fixture and procedure for coating a rod with a fluid film and orienting it into an inclined position.

Behind the rod, across from the camera, a flourescent light is projected through white paper; this produces an even lighting background from which the shadow of the rod and fluid film is sharply visible. An Arduino microcontroller was used to control actuation of the stepper and servo motors. A BasleracA2500-14gc CCD camera (2592x1944 pixels) connected with LabView, was used to acquire images at 10 frames per second. Periodic patterns of droplets form and move along the fibers if they are inclined. Frames from the videos showing the coating patterns for various inclination angles are shown in Figure 3. A corresponding simulation result is shown in Figure 4.

The upper and lower liquid edges can be detected from the experimental videos and compared with the numerical prediction. The upper and lower moving wave patterns are seen to be in phase, both experimentally and in the simulation. Unless the fiber is held horizontally, the lower edge wave amplitude will be greater because of the cross fiber drainage.

The wave speed and calculated average drainage speed $\bar{U}$ are shown in Figure 6 as a function of inclination angle. The average speed or drainage rate on circular cylinders, or, by inference, cylinders of any other cross section, can be increased significantly by inclining the cylinders away from a vertical orientation. This is due to the formation of a thick pendant edge along the underside of an inclined cylinder. Because the local axial flux is proportional to the cube of the coating thickness,

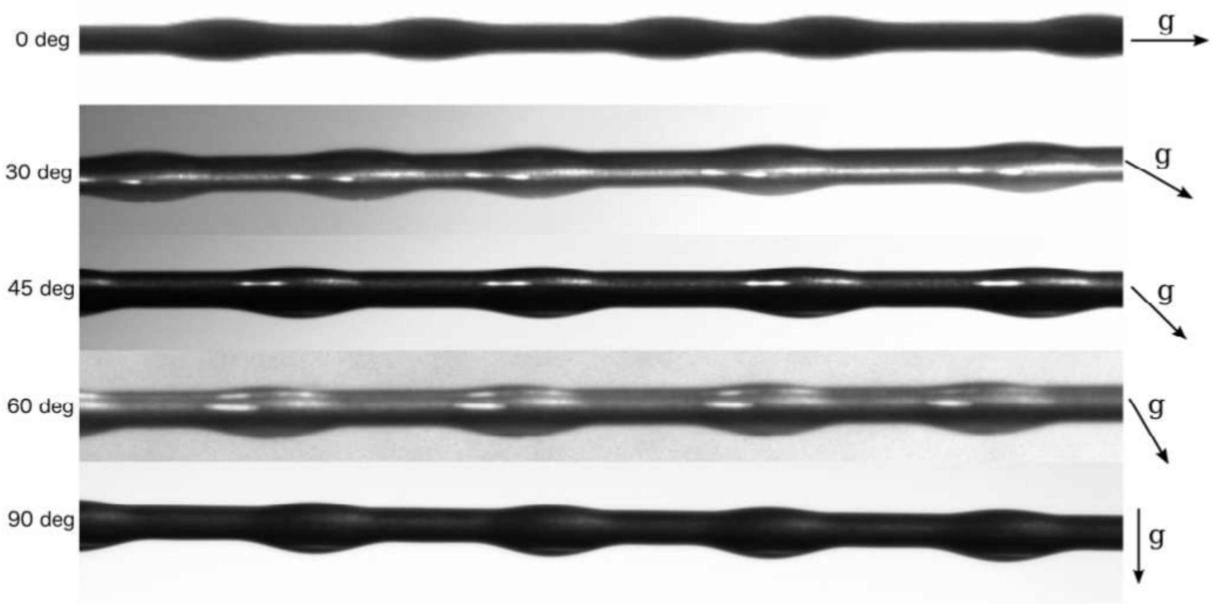


Figure 3: Patterns of fully-developed drops observed experimentally for 0.5 mm diameter fiber with initial coating thickness of h_0/R = 0.16 oriented at 0, 30, 45, 60, and 90 degrees from vertical. The gravity direction is shown on the right edge of the figure.

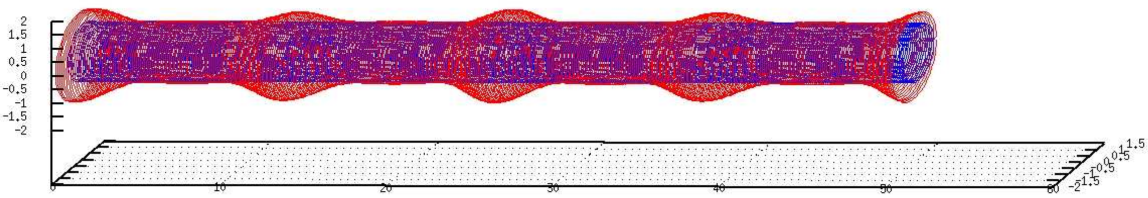
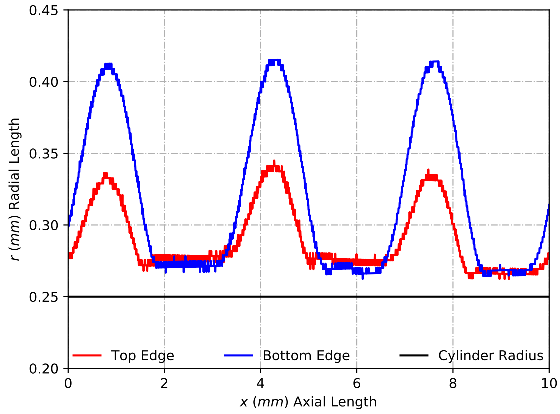


Figure 4: Calulated wire-cage picture corresponding to the horizontal picture above. The cylindrical substrate is visible through the free surface. The cross-sectional scale is exaggerated.

the pendant is much more effective than a uniform coating. Simulation results show that instability-induced droplet formation increases the flow rate still further. Overtaking and coalescing droplets will also increase the transport; however droplets can fall off if they exceed a critical size.



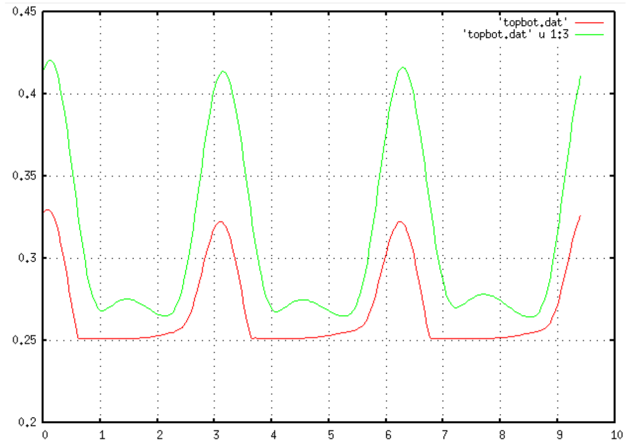


Figure 5: Comparison of experimental and calculated wave profiles. (Left) Experimental result for a 0.5 mm diameter fiber with initial coating thickness of h_0/R = 0.16 oriented at 45 degrees. The upper and lower edges were tracked over time as the fluid instability grew. (Right) Corresponding simulation result. All scales are in mm.

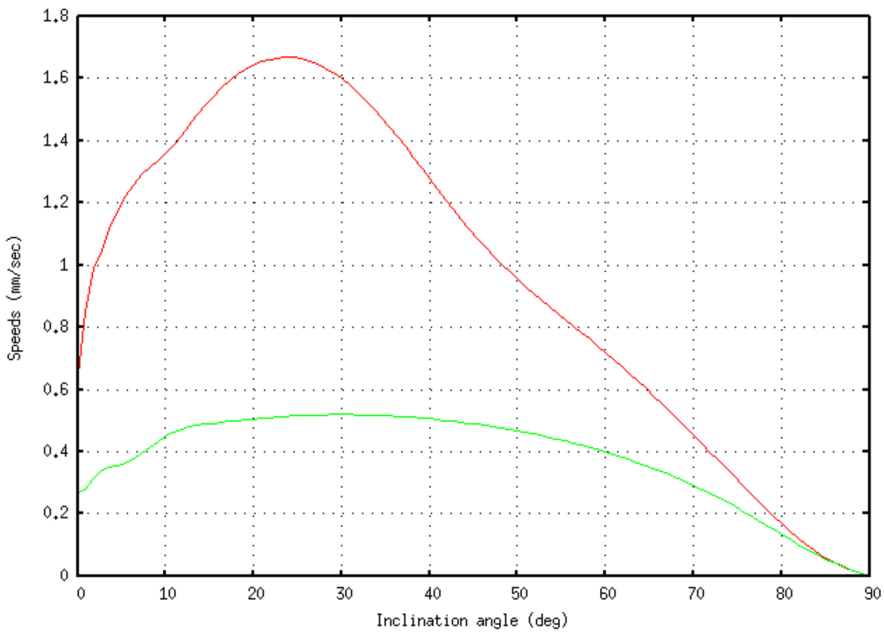


Figure 6: Under side (ϕ = 180^o) wave speed versus inclination angle α in degrees. Simulation results for $2R$ = 1 mm and $B = \rho g R^2/\sigma$ = 0.10. The lower curve is average particle speed $\bar{U}$.