Thermocapillary Flows on Heated Uneven Substrates

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Introduction

When a temperature difference occurs along a gas/liquid interface, a surface tension gradient is generated, which causes static equilibrium impossible. Thermocapillary flows are driven from a low-surface-tension region to a high-surface-tension region. Such flows are observed in several systems, including combustion processes, welding pools, crystal-growth processes, droplet coalescence and formation, etc. In this work, we consider liquid layers on heated substrates, which can be encountered in liquid film coatings. A flat substrate has been used for investigating thermocapillary flows, but only a few studies consider that over uneven substrates.

Asymptotic theory was rarely used to study such flows under a liquid layer over an uneven substrate. We defined flow conditions different from the previous studies, and it results in different asymptotic solutions. Moreover, the conditions to ensure the stability are chosen so that the solutions can directly reflect how the heat and flow behaviors depend on system parameters including substrate topography.

The system is simplified as two-dimensional liquid layer over the uneven substrate, which is described by a sinusoidal wave. The liquid layer is heated below from the substrate and is cooled by the gas. A thickness of the liquid layer varies along the horizontal direction. The temperature at the interface above the trough of the substrate is low because the distance between the interface and the substrate is far. On the contrary, the interface above the crest has a relatively high temperature. Therefore, the resulted temperature gradient along the interface causes the surface tension gradient. The surface tension gradient drives thermocapillary flows, where a vortex exists in a unit cell.

In the present work, the buoyancy force inside the layer is negligible, and the flows are caused by the thermocapillary force along the interface. The liquid layer is, however, relatively thick in order not to allow interactions between the interface and the substrate, such as disjoining pressure. Moreover, low Marangoni and capillary numbers are considered so that the interface deformation is insignificant. We obtained second-order solutions by using a regular perturbation theory to investigate the heat transfer and fluid flows inside the liquid layer over the sinusoidal substrate.

Mathematical description

The sinusoidal substrate is defined by $f(x') = h\sin(2\pi x'/\lambda)$, where λ is the wavelength and h is the amplitude. The position of the interface is $y' = l + d'$. Herein, the mean layer depth, when the interface is flat, is defined by l, and the disturbance from the flat interface is defined by d'. A unit cell is a region bounded by $x' = n\lambda - \lambda/4$ and $x' = n\lambda + \lambda/4$, where n is an arbitrary integer. The aspect ratio of the unit cell is $A = 2\pi\, d/\lambda$, and the amplitude ratio is $H = h/d$.

Scaling by dimensionless variables $x = x'/(\lambda/2\pi)$, $y = y'/l$, $u = u'/U$, $v = v'/AU$, $T = (T' - T^{i'})/\triangle T^{i'}$, the system is governed by

$$Ma\, A\left(u\, T_x + v\, T_y\right) = A^2 T_{xx} + T_{yy},$$

$$Re\, A\left(u\, w_x + v\, w_y\right) = A^2 w_{xx} + w_{yy},$$

$$w = -\psi_{yy} - A^2\psi_{xx}.$$

Introducing the vorticity w and the stream function ψ, we used the vorticity-stream function formulation to describe the momentum transfer. The characteristic velocity $U = -\sigma_T \triangle T^{i'}/\mu$, where σ_T is the thermal coefficient of surface tension; $\triangle T^{i'}$ is the characteristic temperature difference along the interface, and $T^{i'}$ is the characteristic interfacial temperature. The Marangoni number, Reynolds number, Prandtl number, capillary number, and Biot number are defined by

$$Ma = \frac{Ul}{\kappa}, Re = \frac{Ul}{\nu}, Pr = \frac{\nu}{\kappa}, Ca = \frac{\mu U}{\sigma_0}, Bi = \frac{h_i l}{k_l}.$$

In this study, we considered the orders of $Ma = O(A), Re = O(A), Pr = 1, Ca = O(A^6), Bi = O(1)$ and $H = O(0.1)$.

The boundary conditions are

$$\frac{T_y - A^2 d_x T_x}{(1 + A^2 d_x^2)^{\frac{1}{2}}} = -Bi\, T - \frac{1}{2H} \quad \text{on} \quad y = 1 + d,$$

$$T = \frac{1}{2H} \quad \text{on} \quad y = f(x),$$

$$-p + \frac{2A^2}{(1 + A^2 d_x^2)}\left[\left(v_y - d_x u_y\right) + A^2 d_x(-v_x + d_x u_x)\right] = \frac{A^2 d_{xx}}{\left(1 + A^2 d_x^2\right)^{\frac{3}{2}}}\left(\frac{A}{Ca} - T\right) \quad \text{on} \quad y = 1 + d,$$

$$\left(u_y + A^2 v_x\right)(1 - A^2 d_x^2) + 2A^2\left(v_y - u_x\right) = -(1 + A^2 d_x^2)^{1/2}\left(T_x - d_x T_y\right) \quad \text{on} \quad y = 1 + d,$$

$$\psi = \psi_y = 0 \quad \text{on} \quad y = f(x),$$

and no net flow condition becomes

$$\psi = 0 \quad \text{on} \quad y = 1 + d.$$

In a limit of a small aspect ratio $A \to 0$, the asymptotic power series are defined by

$$T = T_0 + AT_1 + A^2T_2 + \cdots$$

$$w = w_0 + Aw_1 + A^2w_2 + \cdots$$

$$\psi = \psi_0 + A\psi_1 + A^2\psi_2 + \cdots$$

$$p = p_0 + Ap_1 + A^2p_2 + \cdots$$

$$d = Ad_1 + A^2d_2 + \cdots$$

Since $Ca = O(A^6)$, $d_1 = d_2 = 0$ is trivial, which the deformation of the interface vanishes in the second-order solutions.

Asymptotic solutions

The basic solution T_0 linearly decreases in the vertical direction and changes with following the sinusoidal profile in the horizontal direction, as shown in figure 1. This temperature gradient along the interface causes a counter-clockwise rotating vortex. Owing to the order of Marangoni and Reynolds numbers, the first corrections are trivial. The convection terms appear starting from the second corrections.

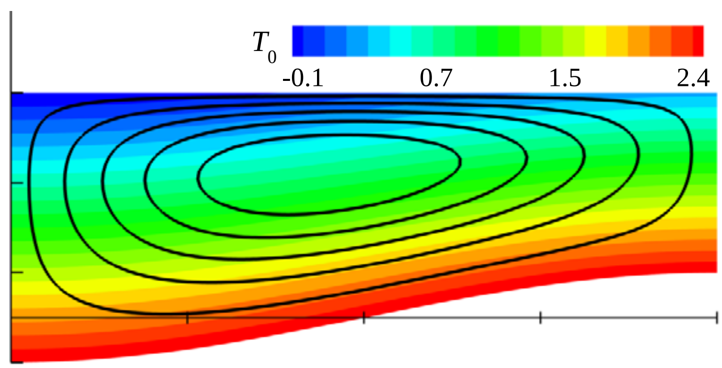

Figure 1. Color contour of $\boldsymbol{T_0}$ and streamlines corresponding to $\boldsymbol{\psi_0}$ when $\boldsymbol{H = 0.2}$ and $\boldsymbol{Bi = 0.1}$. The solid line represents a counter-clockwise rotation.

We can split as $T_2 = T_2^{\mathrm{D}} + T_2^{\mathrm{C}}$ by the linear combination of two parts, where superscripts D and C denote that the quantities come from the horizontal diffusion and convection of T_0, respectively. T_2^{C} and corresponding temperature gradient along the interface intensify the overall flows, as shown in figure 2.

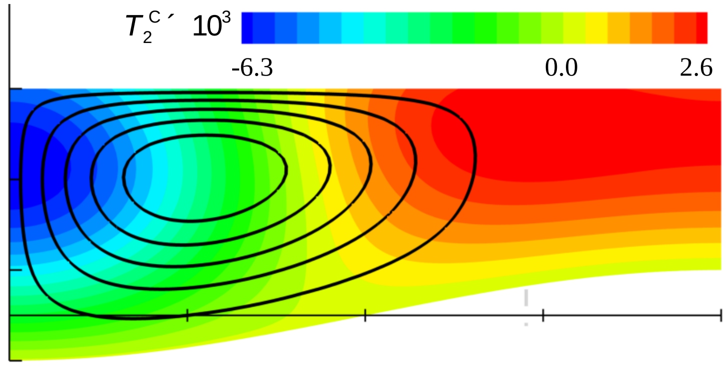

Figure 2. Color contour of $\boldsymbol{T_2^{\mathrm{C}}}$ and streamlines corresponding to $\boldsymbol{\psi_2^{\mathrm{hC}}}$ when $\boldsymbol{H} = \mathbf{0.2}$ and $\boldsymbol{Bi} = \mathbf{0.1}$. $\boldsymbol{\psi_2^{\mathrm{hC}}}$ is a part of homogeneous solutions in second-order correction for momentum transfer.

Apart from that, T_2^{D} shows a different aspect comparing to T_0 with respect to the direction of the temperature gradient. The change in the direction of the temperature gradient causes a clockwise rotating vortex, as shown in figure 3. Those behaviors arise from the horizontal heat diffusion $-T_{0,xx}$.

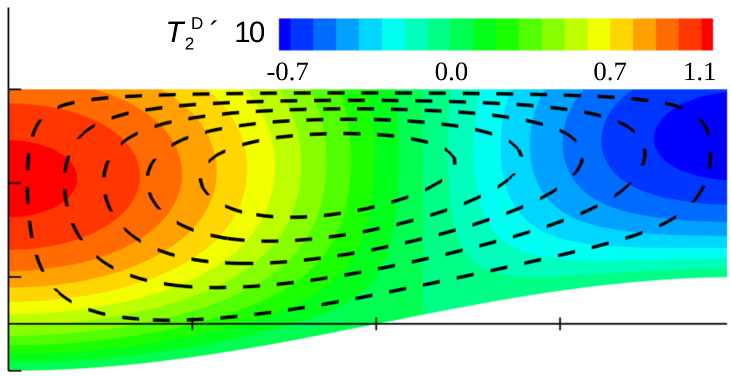

Figure 3. Color contour of $\boldsymbol{T_2^{\mathrm{D}}}$ and streamlines corresponding to $\boldsymbol{\psi_2^{\mathrm{hD}}}$ when $\boldsymbol{H} = \mathbf{0.2}$ and $\boldsymbol{Bi} = \mathbf{0.1}$. $\boldsymbol{\psi_2^{\mathrm{hD}}}$ is a part of homogeneous solutions in second-order correction for momentum transfer. The dotted line represents a clockwise rotation.

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